

Math 132 - Quiz 1

January 20, 2021

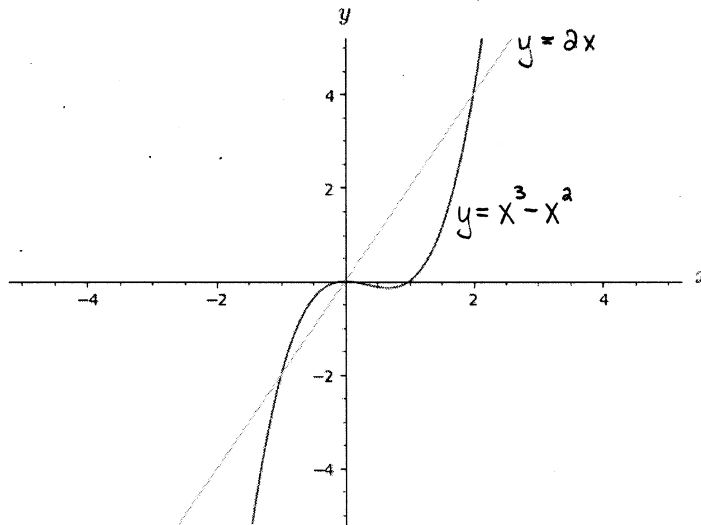
Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due January 27.

- (4 points) Find the total combined area of the bounded regions between the graphs of $y = x^3 - x^2$ and $y = 2x$.

The graphs of the functions are shown here.



$$x^3 - x^2 = 2x$$

$$x^3 - x^2 - 2x = 0$$

$$x(x-2)(x+1) = 0$$

$$x = 0, x = 2, x = -1$$

$$\left[0 - \left(\frac{1}{4} + \frac{1}{3} - 1 \right) \right]$$

$$+ \left[\left(4 - 4 + \frac{8}{3} \right) - 0 \right]$$

TOTAL AREA = AREA OF LEFT REGION
+ AREA OF RIGHT REGION

$$= \boxed{\frac{37}{12}}$$

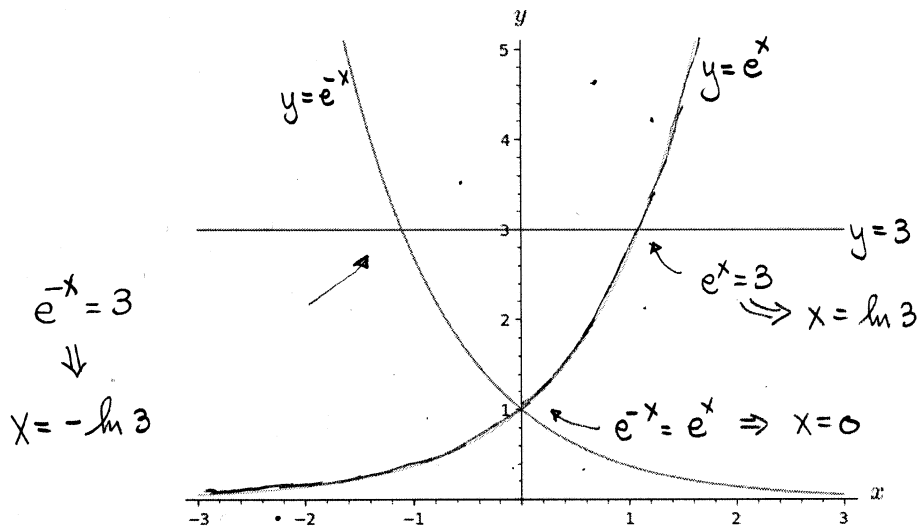
$$= \int_{-1}^0 (x^3 - x^2 - 2x) dx + \int_0^2 (2x - x^3 + x^2) dx$$

$$= \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \right) \Big|_{-1}^0 + \left(x^2 - \frac{1}{4}x^4 + \frac{1}{3}x^3 \right) \Big|_0^2$$

Turn over.

2. (3 points) Find the area of the bounded region enclosed by the graphs of $y = 3$, $y = e^x$, and $y = e^{-x}$.

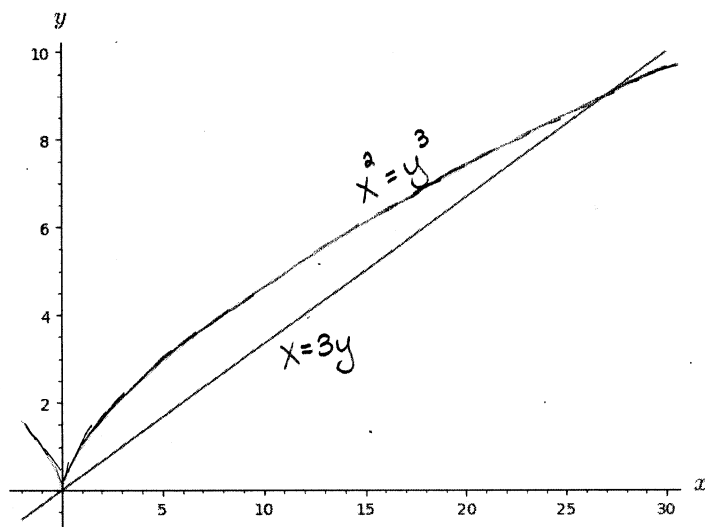
The graphs are shown here.



$$\begin{aligned}
 \text{Area} &= \int_{-\ln 3}^0 (3 - e^{-x}) dx + \int_0^{\ln 3} (3 - e^x) dx \\
 &= 2 \int_0^{\ln 3} (3 - e^x) dx \\
 &= 2 \left(3x - e^x \right) \Big|_0^{\ln 3} \\
 &= 2(3 \ln 3 - 3) - 2(0 - 1) \\
 &= \boxed{6 \ln 3 - 4}
 \end{aligned}$$

3. (3 points) Find the area of the 1st-quadrant region between the graphs of $x^2 = y^3$ and $x = 3y$.

The graphs are shown here.



$$\begin{aligned}
 \text{Area} &= \int_0^9 (3y - y^{3/2}) dy \\
 &= \left(\frac{3}{2} y^2 - \frac{2}{5} y^{5/2} \right) \Big|_0^9 \\
 &= \left(\frac{243}{2} - \frac{486}{5} \right) - 0 \\
 &= \boxed{\frac{243}{10}}
 \end{aligned}$$

$$(3y)^2 = y^3 \Rightarrow 9y^2 - y^3 = 0$$

$$\Rightarrow y^2(9 - y) = 0$$

$$y = 0, y = 9$$

Also could use

$$\int_0^{27} \left(x^{2/3} - \frac{x}{3} \right) dx.$$