

Math 132 - Quiz 2

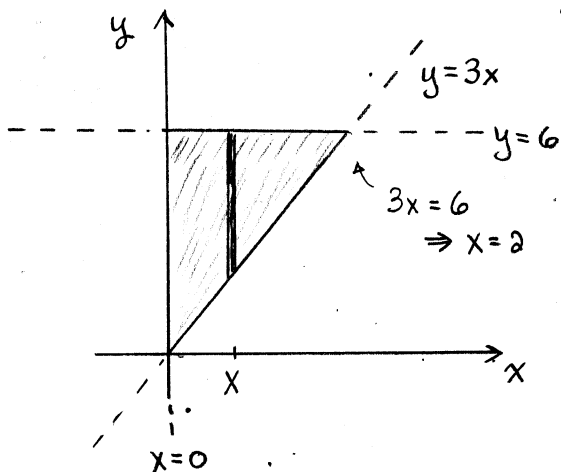
January 27, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due February 3.

1. (2.5 points) The base of a solid is the region bounded by the graphs of $y = 3x$, $y = 6$, and $x = 0$. The cross sections perpendicular to the x -axis are rectangles of constant height 10. Find the volume of the solid.



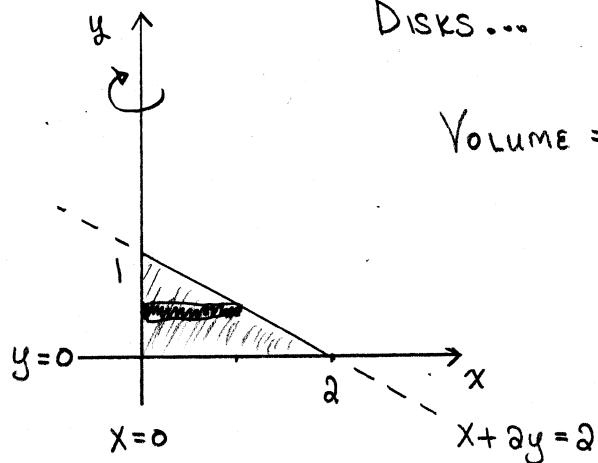
AREA OF CROSS SECTION AT X

$$\begin{aligned} &= (\text{HEIGHT AT } X)(\text{LENGTH AT } X) \\ &= (10)(6 - 3x) \end{aligned}$$

$$\text{VOLUME} = \int_0^2 (60 - 30x) dx$$

$$= 60x - 15x^2 \Big|_0^2 = \boxed{60}$$

2. (2.5 points) The region bounded by the graphs of $x + 2y = 2$, $x = 0$, and $y = 0$ is rotated about the y -axis. Find the volume of the solid that is generated.



DISKS ...

$$\text{VOLUME} = \pi \int_0^1 (2 - 2y)^2 dy$$

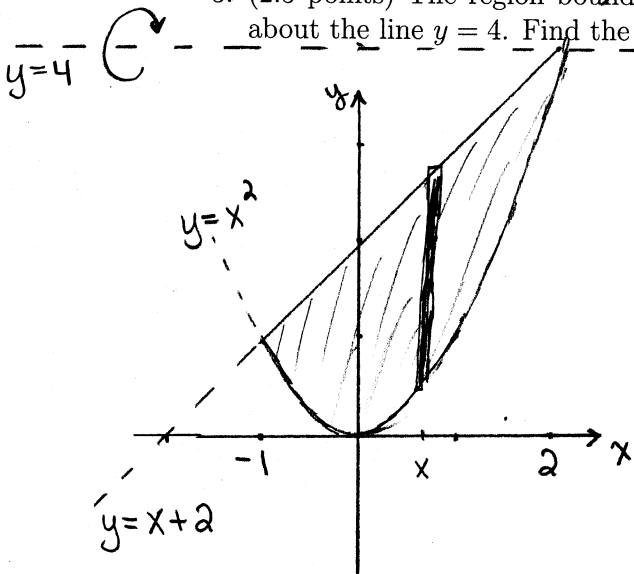
$$= \pi \int_0^1 (4 - 8y + 4y^2) dy$$

$$= \pi \left(4y - 4y^2 + \frac{4}{3}y^3 \right) \Big|_0^1$$

Turn over.

$$= \boxed{\frac{4}{3} \pi}$$

3. (2.5 points) The region bounded by the graphs of $y = x^2$ and $y = x + 2$ is rotated about the line $y = 4$. Find the volume of the solid that is generated.



$$x^2 = x + 2$$

$$x^2 - x - 2 = (x - 2)(x + 1) = 0$$

$$x = 2, x = -1$$

WASHERS...

$$\text{Volume} = \pi \int_{-1}^2 (4 - x^2)^2 - (4 - (x + 2))^2 dx$$

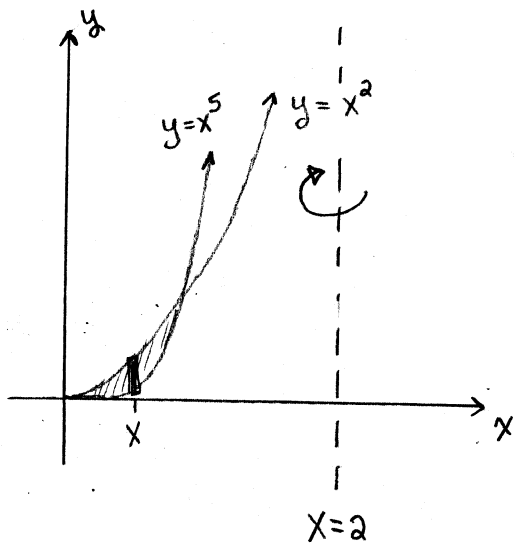
$$= \pi \int_{-1}^2 (16 - 8x^2 + x^4 - (4 - 4x + x^2)) dx$$

$$= \pi \int_{-1}^2 (12 + 4x - 9x^2 + x^4) dx =$$

$$= \pi \left(12x + 2x^2 - 3x^3 + \frac{1}{5}x^5 \right) \Big|_{-1}^2 = \pi \left(\frac{72}{5} + \frac{36}{5} \right)$$

$$= \frac{108}{5} \pi$$

4. (2.5 points) The 1st-quadrant region bounded by the graphs of $y = x^2$ and $y = x^5$ is rotated about the line $x = 2$. Find the volume of the solid that is generated.



SHELLS...

$$\text{Volume} = 2\pi \int_0^1 (2 - x)(x^2 - x^5) dx$$

$$= 2\pi \int_0^1 (2x^2 - 2x^5 - x^3 + x^6) dx$$

$$= 2\pi \left(\frac{2}{3}x^3 - \frac{1}{3}x^6 - \frac{1}{4}x^4 + \frac{1}{7}x^7 \right) \Big|_0^1$$

$$= 2\pi \left(\frac{2}{3} - \frac{1}{3} - \frac{1}{4} + \frac{1}{7} \right)$$

$$= 2\pi \left(\frac{19}{84} \right) = \frac{19\pi}{42}$$

$$x^2 = x^5$$

↓

$$x^2(x^3 - 1) = 0$$

$$x = 0, x = 1$$