Math 132 - Quiz 3

February 24, 2021

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 3.

1. (2 points) Use integration by parts to evaluate $\int \cos(5t)\,\sin(6t)\,dt$.

Signs	u & derivs	dv/dx & antis
+	$\cos(5t)$	$\sin(6t)$
−	$-5\sin(5t)$	$-\frac{1}{6}\cos(6t)$
+	$-25\cos(5t)$	$-\frac{1}{36}\sin 6t$

Recurring integral

$$\int \cos(5t)\sin(6t)\,dt = -\frac{1}{6}\cos(5t)\cos(6t) - \frac{5}{36}\sin(5t)\sin(6t) + \frac{25}{36}\int \cos(5t)\sin(6t)\,dt$$

$$\boxed{\int \cos(5t)\sin(6t)\,dt = \frac{36}{11}\left(-\frac{1}{6}\cos(5t)\cos(6t) - \frac{5}{36}\sin(5t)\sin(6t)\right) + C}$$

2. (2 points) Use a product-to-sum trig identity to evaluate $\int \cos(5t)\,\sin(6t)\,dt$.

$$\cos(5t)\sin(6t) = \frac{1}{2}\left[\sin(11t) + \sin(t)\right]$$

$$\frac{1}{2}\int \left[\sin(11t) + \sin(t)\right] dt$$

$$= \boxed{-\frac{1}{22}\cos(11t) - \frac{1}{2}\cos(t) + C}$$

Turn over.

3. (2 point) Integrate: $\displaystyle\int_0^1 (2x^2 + 3x)\, e^{5x}\, dx$

Signs	u & derivs	dv/dx & antis
$+$	$2x^2+3x$	e^{5x}
$-$	$4x+3$	$\frac{1}{5}e^{5x}$
$+$	4	$\frac{1}{25}e^{5x}$
$-$	0	$\frac{1}{125}e^{5x}$

$$= \frac{1}{5}(2x^2+3x)e^{5x} - \frac{1}{25}(4x+3)e^{5x} + \frac{4}{125}e^{5x}\Big|_0^1$$

$$= e^5\left[\frac{5}{5} - \frac{7}{25} + \frac{4}{125}\right] - \left[0 - \frac{3}{25} + \frac{4}{125}\right]$$

$$= \boxed{\frac{94}{125}e^5 + \frac{11}{125}}$$

4. (2 points) Integrate: $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}\, dx$

$$\int \frac{\cos^4 x \cos x}{\sqrt{\sin x}}\, dx = \int \frac{(1-\sin^2 x)^2 \cos x\, dx}{\sqrt{\sin x}} = \int \frac{(1-u^2)^2\, du}{u^{1/2}}$$

$$u = \sin x, \quad du = \cos x\, dx$$

$$= \int \left(u^{-1/2} - 2u^{3/2} + u^{7/2}\right) du = 2u^{1/2} - \frac{4}{5}u^{5/2} + \frac{2}{9}u^{9/2} + C$$

$$= \boxed{2(\sin x)^{1/2} - \frac{4}{5}(\sin x)^{5/2} + \frac{2}{9}(\sin x)^{9/2} + C}$$

5. (2 points) Integrate: $\displaystyle\int \tan^3 x \sec^4 x\, dx$

$$\int \tan^3 x \sec^2 x \sec^2 x\, dx = \int \tan^3 x\,(1+\tan^2 x)\sec^2 x\, dx$$

$$u = \tan x, \quad du = \sec^2 x\, dx$$

$$= \int u^3(1+u^2)\, du = \int (u^3 + u^5)\, du = \frac{1}{4}u^4 + \frac{1}{6}u^6 + C$$

$$= \boxed{\frac{1}{4}(\tan x)^4 + \frac{1}{6}(\tan x)^6 + C}$$