Math 132 - Quiz 4

March 24, 2021

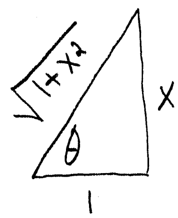
Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due March 31.

1. (3 points) Use a trigonometric substitution to evaluate $\displaystyle\int \frac{x^2}{\sqrt{1+x^2}}\,dx$.

$x = \tan\theta,\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$dx = \sec^2\theta\, d\theta$

$$\int \frac{\tan^2\theta \sec^2\theta\, d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{\tan^2\theta\sec^2\theta}{\sqrt{\sec^2\theta}}\,d\theta$$

$$= \int \frac{\tan^2\theta \sec^2\theta\, d\theta}{|\sec\theta|} = \int \tan^2\theta \sec\theta\, d\theta$$

$|\sec\theta| = \sec\theta$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

See key for HW 3.

$$= \int (\sec^3\theta - \sec\theta)\, d\theta = \int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta$$

$$= \left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right] - \ln|\sec\theta + \tan\theta| + C$$

$$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\frac{1}{2}\sqrt{1+x^2}\,x - \frac{1}{2}\ln\left|\sqrt{1+x^2} + x\right| + C}$$

Turn over.

2. (3 points) Use partial fractions to evaluate $\int \frac{3x+4}{(x^2+4)(3-x)}\,dx$.

$$\frac{3x+4}{(x^2+4)(3-x)} = \frac{A}{3-x} + \frac{Bx+C}{x^2+4}$$

$$3x+4 = A(x^2+4) + (Bx+C)(3-x)$$

$$x=3: \quad 13 = 13A \Rightarrow A = 1$$

$$x=0: \quad 4 = 4A + 3C = 4 + 3C \Rightarrow C = 0$$

$$x=1: \quad 7 = 5A + (B+C)(2)$$
$$= 5 + 2B \Rightarrow B = 1$$

$$\int \frac{1}{3-x}\,dx + \int \frac{x}{x^2+4}\,dx$$

$u = 3-x$ $\qquad$ $w = x^2+4$

$du = -dx$ $\qquad$ $dw = 2x\,dx$

$$-\int \frac{1}{u}\,du + \frac{1}{2}\int \frac{1}{w}\,dw$$

$$-\ln|u| + \frac{1}{2}\ln|w| + C = \boxed{-\ln|3-x| + \frac{1}{2}\ln(x^2+4) + C}$$

3. (2 points) Use the trapezoid rule over 5 subintervals to approximate $\int_1^2 \sin(x^2)\, dx$.

$$h = \frac{2-1}{5} = \frac{1}{5}$$

PARTITION: $1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2$

$$T = \frac{1/5}{2}\left[\sin(1) + 2\sin\left(\frac{36}{25}\right) + 2\sin\left(\frac{49}{25}\right) + 2\sin\left(\frac{64}{25}\right) + 2\sin\left(\frac{81}{25}\right) + \sin(4)\right]$$

$$\approx 0.4820222$$

4. (2 points) Evaluate the improper integral $\int_0^\infty \frac{e^x}{1+e^{2x}}\, dx$.

$$\lim_{t\to\infty} \int_0^t \frac{e^x}{1+e^{2x}}\, dx = \lim_{t\to\infty} \int_1^{e^t} \frac{1}{1+u^2}\, du$$

$u = e^x$

$du = e^x\, dx$

$$= \lim_{t\to\infty} \tan^{-1} u \Big|_1^{e^t}$$

$$= \lim_{t\to\infty} \left(\tan^{-1} e^t - \tan^{-1} 1\right)$$

$$= \lim_{t\to\infty} \tan^{-1} e^t - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$