Math 132 - Quiz 5

April 7, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 14.

1. (2 points) Use a partial fraction decomposition to rewrite the terms of the series. Then determine if the series converges or diverges. If it converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{2}{4n^2-1} = \sum_{n=1}^{\infty} \frac{1}{2n-1} - \frac{1}{2n+1}$$

$$= \left(1 - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right) + \cdots$$

$$\frac{2}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$$2 = A(2n+1) + B(2n-1)$$

$n = \frac{1}{2}$: $2 = 2A \Rightarrow A = 1$

$n = -\frac{1}{2}$: $2 = -2B \Rightarrow B = -1$

$$S_n = 1 - \frac{1}{2n+1}$$

$$S_n \to \boxed{1} \text{ AS } n \to \infty$$

SERIES CONVERGES. Sum = 1.

2. (2 points) Determine whether the series converges or diverges. If it converges, find the sum.

$$\sum_{n=2}^{\infty} \frac{11}{4^n}$$

$$\sum_{n=2}^{\infty} 11 \left(\frac{1}{4}\right)^n$$

GEOMETRIC w/ $r = \frac{1}{4}$

CONVERGES

$$\sum_{n=0}^{\infty} 11\left(\frac{1}{4}\right)^n = \frac{11}{1-\frac{1}{4}} = \frac{44}{3}$$

$$\Rightarrow \sum_{n=2}^{\infty} 11\left(\frac{1}{4}\right)^n = \frac{44}{3} - 11 - \frac{11}{4}$$

$$= \boxed{\frac{11}{12}}$$

Turn over.

3. (6 points) Determine whether each series converges or diverges. Be sure to show work and/or explain your reasoning.

(a) $\sum_{n=0}^{\infty} \tan^{-1} n$

$$\lim_{n\to\infty} \text{TAN}^{-1} n = \frac{\pi}{2} \neq 0$$

SERIES DIVERGES BY n^{TH} TERM TEST.

(b) $\sum_{n=0}^{\infty} \frac{1}{n^2+4}$

COMPARE WITH $\sum \frac{1}{n^2}$, $p=2$, CONVERGENT.

$$\lim_{n\to\infty} \frac{\frac{1}{n^2+4}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+4} = \lim_{n\to\infty} \frac{1}{1+\frac{4}{n^2}} = 1$$

SERIES CONVERGES BY LIMIT COMPARISON.

(c) $\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^7+5}}$

COMPARE WITH $\sum \frac{1}{\sqrt{n}}$, $p=\frac{1}{2}$, DIVERGENT.

$$\lim_{n\to\infty} \frac{\frac{n^3}{\sqrt{n^7+5}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{n^3\sqrt{n}}{\sqrt{n^7}+5} = \lim_{n\to\infty} \frac{n^{7/2}}{n^{7/2}+5}$$

$$= \lim_{n\to\infty} \frac{1}{1+\frac{5}{n^{7/2}}} = 1$$

SERIES DIVERGES BY LIMIT COMPARISON.