**Math 132 - Quiz 6**
April 21, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due April 28.

1. (4 points) Determine whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

RATIO TEST...

$$\lim_{n\to\infty} \left| \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} \right| = \lim_{n\to\infty} \frac{2^{n+1}\, n!}{2^n\,(n+1)!} = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$$

$\boxed{\text{SERIES CONVERGES.}}$

(b) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{4k^2 - 3k}$

COMPARE WITH THE CONVERGENT $p = \frac{3}{2}$ SERIES.

$$\lim_{k\to\infty} \frac{\frac{\sqrt{k}}{4k^2-3k}}{\frac{1}{k^{3/2}}} = \lim_{k\to\infty} \frac{\sqrt{k}\; k^{3/2}}{4k^2-3k} = \lim_{k\to\infty} \frac{k^2}{4k^2-3k}$$

$$= \lim_{k\to\infty} \frac{1}{4 - 3/k} = \frac{1}{4}$$

SERIES CONVERGES BY LIMIT COMPARISON WITH $\sum \frac{1}{n^{3/2}}$

*Turn over.*

2. (4 points) Consider the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$.

(a) Use the alternating series test to show that the series converges.

$a_k = \frac{1}{2k+1}$ HAS LIMIT $0$ AS $k \to \infty$

AND. $0 < a_{k+1} < a_k$ FOR EACH $k$.

SERIES CONVERGES BY AST.

(b) Does the series converge absolutely? Explain why or why not.

$\boxed{\text{No.}}$ THE SERIES CONVERGES CONDITIONALLY.

$\sum_{k=0}^{\infty} \frac{1}{2k+1}$ DIVERGES BY LIMIT COMPARISON WITH $\sum \frac{1}{k}$.

$$\lim_{k\to\infty} \frac{\frac{1}{2k+1}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{k}{2k+1} = \frac{1}{2}.$$

THE SERIES CONVERGES, BUT THE SERIES OF ABS. VALUES DIVERGES.

(c) It was first discovered by Indian mathematicians in the 12th century that the series converges to $\pi/4$. Use the alternating series remainder theorem to find an upper bound on the error made in the approximation

$$\frac{\pi}{4} \approx \underbrace{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}}_{S_5}.$$

$$\left| S_5 - \frac{\pi}{4} \right| < a_6 = \frac{1}{13}$$

3. (2 points) Determine whether the series converges or diverges: $\sum_{n=1}^{\infty} \frac{(-1)^n 5^{3n}}{n^n}$

ROOT TEST...

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)\, 5^{3n}}{n^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{5^3}{n}\right)^n} = \lim_{n\to\infty} \frac{5^3}{n} = 0 < 1$$

SERIES CONVERGES.