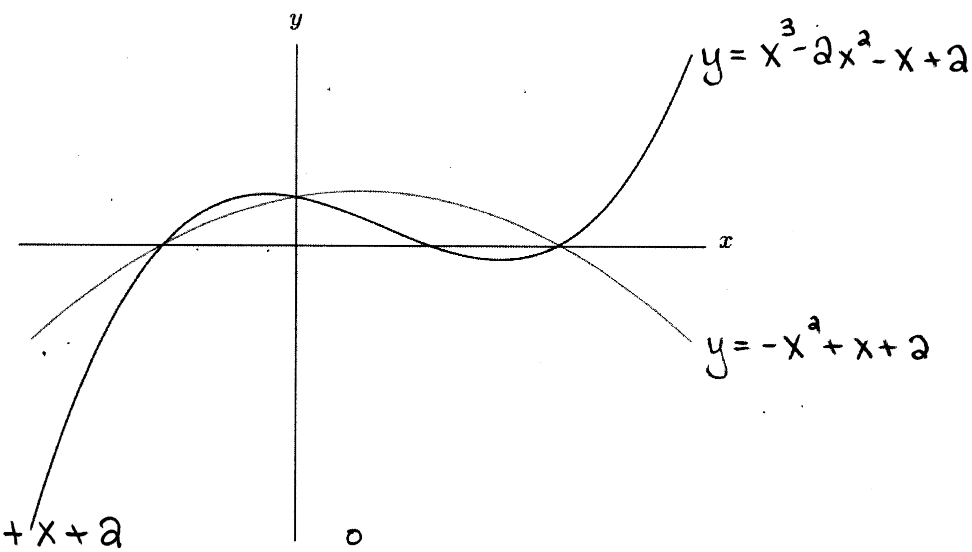
Math 132 - Test 1
February 10, 2021

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, evaluate all integrals by hand. (However, you may check your work with your calculator.)

1. (15 points) The graphs of $y = x^3 - 2x^2 - x + 2$ and $y = -x^2 + x + 2$ are shown below. Find the intersection points. Then compute the total combined area of the bounded regions between the graphs.



$x^3 - 2x^2 - x + 2 = -x^2 + x + 2$

$x^3 - x^2 - 2x = 0$

$x(x-2)(x+1) = 0$

$x = 0, \; x = 2, \; x = -1$

$$\text{AREA} = \int_{-1}^{0} (x^3 - 2x^2 - x + 2) - (-x^2 + x + 2)\, dx + \int_{0}^{2} (-x^2 + x + 2) - (x^3 - 2x^2 - x + 2)\, dx$$

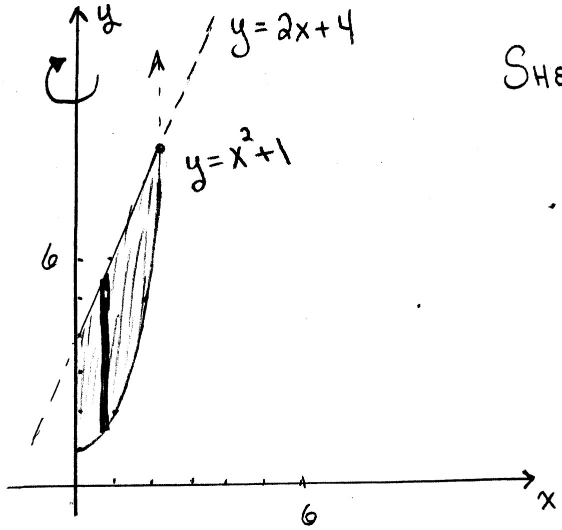
$$= \int_{-1}^{0} (x^3 - x^2 - 2x)\, dx + \int_{0}^{2} (-x^3 + x^2 + 2x)\, dx$$

$$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \Big|_{-1}^{0} + -\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 \Big|_{0}^{2}$$

$$= 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) + \left(-4 + \frac{8}{3} + 4\right) - 0$$

$$= \frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}}$$

2. (15 points) The first quadrant region bounded by the graphs of $y = x^2 + 1$, $y = 2x + 4$, and $x = 0$ is rotated about the y-axis. Find the volume of the solid that is generated.



Shells ...

$$2\pi \int_0^3 x \left[(2x+4) - (x^2+1) \right] dx$$

$$= 2\pi \int_0^3 x \cdot (-x^2 + 2x + 3)\, dx$$

$$= 2\pi \int_0^3 (-x^3 + 2x^2 + 3x)\, dx$$

$$= 2\pi \left(-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{3}{2}x^2 \right) \Big|_0^3$$

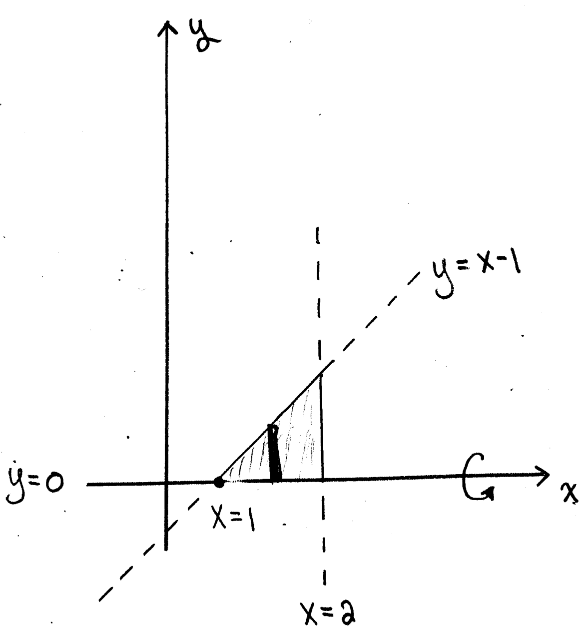
$$= 2\pi \left[\left(-\frac{81}{4} + 18 + \frac{27}{2} \right) - 0 \right] = \boxed{22.5\pi}$$

$x^2 + 1 = 2x + 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0 \quad x = 3,\ x = -1$

3. (12 points) The region bounded by the graphs of $y = x - 1$, $y = 0$, and $x = 2$ is rotated about the x-axis. Find the volume of the solid that is generated.



Disks ...

$$\pi \int_1^2 (x-1)^2\, dx$$

$$= \pi \int_1^2 (x^2 - 2x + 1)\, dx$$

$$= \pi \left(\frac{1}{3}x^3 - x^2 + x \right) \Big|_1^2$$

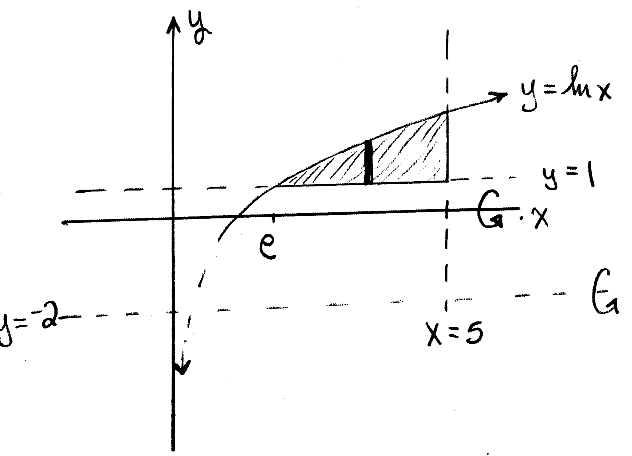
$$= \pi \left[\left(\frac{8}{3} - 4 + 2 \right) - \left(\frac{1}{3} - 1 + 1 \right) \right]$$

$$= \boxed{\frac{\pi}{3}}$$

4. (12 points) The region bounded by the graphs of $y = \ln x$, $y = 1$, and $x = 5$ is rotated about the x-axis to form a solid.

(a) Set up the definite integral that gives the volume of the solid. DO NOT evaluate the integral.

$\ln x = 1 \Rightarrow x = e$



WASHERS ...

$$\boxed{\pi \int_e^5 \left[(\ln x)^2 - (1)^2 \right] dx}$$

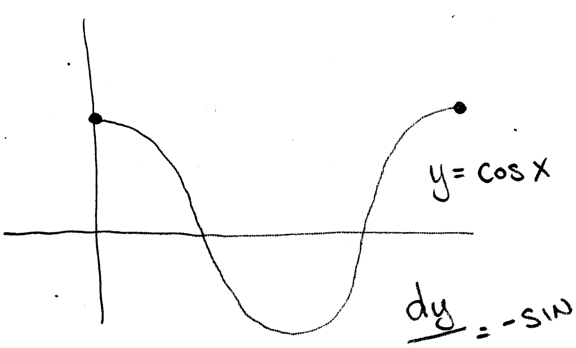
(b) Now rotate the same region about the line $y = -2$. Set up the definite integral that gives the volume of the new solid. DO NOT evaluate the integral.

INSIDE AND OUTSIDE RADII ARE 2 UNITS BIGGER.

WASHERS ...

$$\boxed{\pi \int_e^5 \left[(2 + \ln x)^2 - (3)^2 \right] dx}$$

5. (8 points) Find the length of the graph of $y = \cos x$ from the point where $x = 0$ to the point where $x = 2\pi$. Set up the appropriate definite integral, and then use your calculator to approximate its value.

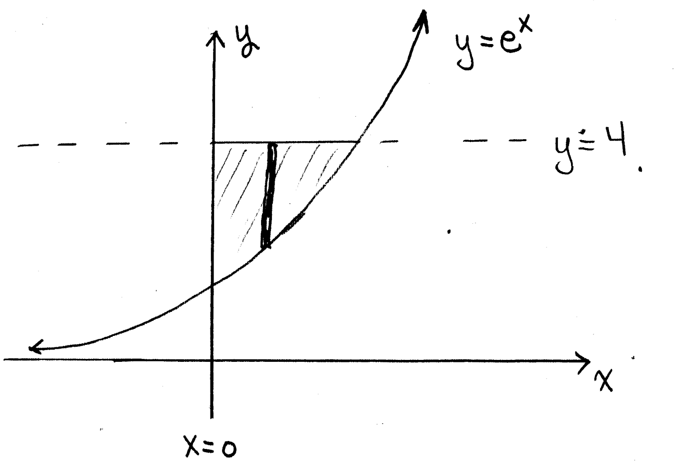


$\frac{dy}{dx} = -\sin x$

$$\int_0^{2\pi} \sqrt{1 + (-\sin x)^2}\, dx$$

$$= \boxed{\int_0^{2\pi} \sqrt{1 + \sin^2 x}\, dx \approx 7.64}$$

6. (15 points) The base of a solid is the first quadrant region bounded by the graphs of $y = e^x$, $y = 4$, and $x = 0$. Each cross section perpendicular to the x-axis is a rectangle whose height is twice the base length. Find the volume of the solid.



$e^x = 4 \Rightarrow x = \ln 4$

AREA OF CROSS SECTION AT X

$= \text{base} \times \text{height}$

$= (4-e^x)(2)(4-e^x)$

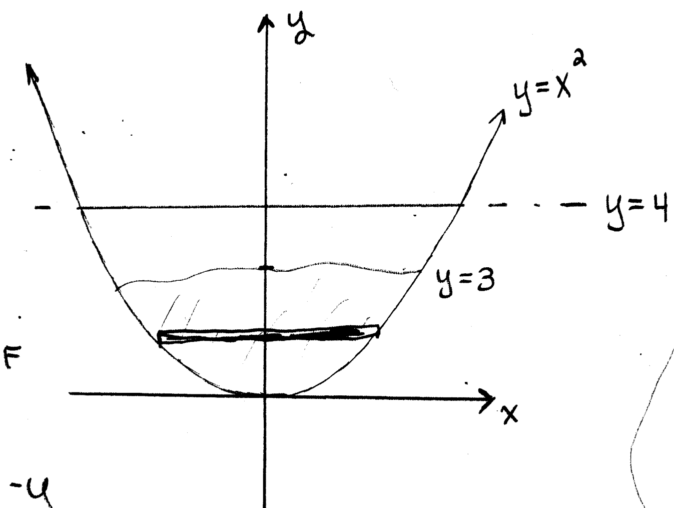
$= 32 - 16e^x + 2e^{2x}$

$$V = \int_0^{\ln 4} (32 - 16e^x + 2e^{2x})\,dx$$

$$= 32x - 16e^x + e^{2x} \Big|_0^{\ln 4}$$

$$= \left[(32\ln 4) - 16(4) + 16\right] - \left[-16+1\right] = \boxed{32\ln 4 - 33 \approx 11.36}$$

7. (8 points) The end of a large trough is a metal plate in the shape of the parabola $y = x^2$. From bottom (vertex) to top (horizontal line), the plate is 4 feet tall. The trough that will hold water weighing $64\,\text{lb/ft}^3$. Find the fluid force on the plate when the water in the trough is 3 feet deep. Set up, but DO NOT evaluate, the required definite integral.



DEPTH OF STRIP $= 3-y$

FLUID FORCE =

$$64\int_0^3 (3-y)(2)(\sqrt{y})\,dy$$

LENGTH OF STRIP

$= \sqrt{y} - (-\sqrt{y})$

$= 2\sqrt{y}$

8. (15 points) For each problem below, think about the region bounded by the graphs of $y = x^3$ and $y = x^2$. Say what is wrong with each result and supply a correction. DO NOT evaluate any of the integrals.

(a) The area of the bounded region between the graphs is given by $A = \int_0^1 (x^3 - x^2)\,dx$.

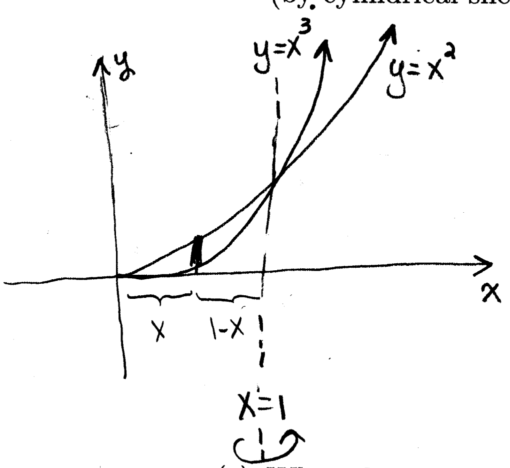
THE SUBTRACTION IS IN THE WRONG ORDER.

TOP CURVE − BOTTOM CURVE $= X^2 - X^3$

CORRECTION: $A = \int_0^1 (x^2 - x^3)\,dx$

(b) When the region is rotated about the line $x = 1$, the volume of the resulting solid (by cylindrical shells) is given $V = 2\pi \int_0^1 x(x^2 - x^3)\,dx$.

DISTANCE FROM STRIP TO AXIS OF ROTATION IS $1-X$



CORRECTION:

$$V = 2\pi \int_0^1 (1-x)(x^2 - x^3)\,dx$$

(c) When the region is rotated about the x-axis, the volume of the resulting solid (by washers) is given by $V = \pi \int_0^1 (x^2 - x^3)^2\,dx$.

SHOULD BE

$$(\text{OUTER RADIUS})^2 - (\text{INNER RADIUS})^2$$

$$= (x^2)^2 - (x^3)^2$$

CORRECTION:

$$V = \pi \int_0^1 \left[(x^2)^2 - (x^3)^2\right] dx$$