Math 132 - Test 3

April 14, 2021

Name ______________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary. Unless otherwise indicated, evaluate all integrals by hand. (However, you may check your work with your calculator.)

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1. (10 points) Use a trig substitution to evaluate the indefinite integral: $\displaystyle\int \frac{dx}{\sqrt{4+x^2}}$

2. (5 points) Write the form of the partial fraction decomposition, but DO NOT SOLVE for the undetermined coefficients.

$$\frac{x^2+3x-7}{x(3x+5)^2(x^2+7)}$$

3. (11 points) Use partial fractions to evaluate the indefinite integral.

$$\int \frac{3x}{x^2 + 2x - 8}\,dx$$

4. (4 points) Steve thought it might be wise to use a partial fraction decomposition to simplify the integral $\displaystyle\int \frac{x+5}{(x^2+1)^2}\,dx$. What do you think about Steve's strategy?

5. (6 points) Use the trapezoid rule with $n = 5$ to approximate $\displaystyle\int_1^2 \frac{dx}{x}$.

6. (3 points) If you used the trapezoid rule with $n = 1$ to approximate $\displaystyle\int_0^1 (3x+5)\,dx$, you would get the exact value of the integral. Why do you think that is so?

7. (11 points) Write as a limit and evaluate: $\displaystyle\int_{-\infty}^0 xe^x\,dx$

8. (8 points) Write as a limit and evaluate: $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

9. (4 points) Explain why the following integral is improper and describe how you would evaluate it. DO NOT EVALUATE.

$$\int_{-2}^{2} \frac{1}{(1+x)^2}\,dx$$

10. (5 points) Write the first four terms of the sequence $\left\{\dfrac{n^2}{e^n}\right\}_{n=1}^{\infty}$. Then find the limit of the sequence (if it exists).

11. (5 points) Consider the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_1 = 2$ and $a_{n+1} = a_n + 3$ for $n \geq 1$. Write the first four terms of the sequence and find an explicit formula for a_n.

12. (8 points) Use the integral test to determine whether $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges or diverges.

13. (20 points) Determine whether each series converges or diverges. Show worok or explain. If the series converges, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n} - \frac{3}{n+2}\right)$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{7e^n}{\pi^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2+n}$