# Math 132 - Final Exam A

May 7, 2021

Name key

Score

**Show all work to receive full credit.** This test is due May 12.

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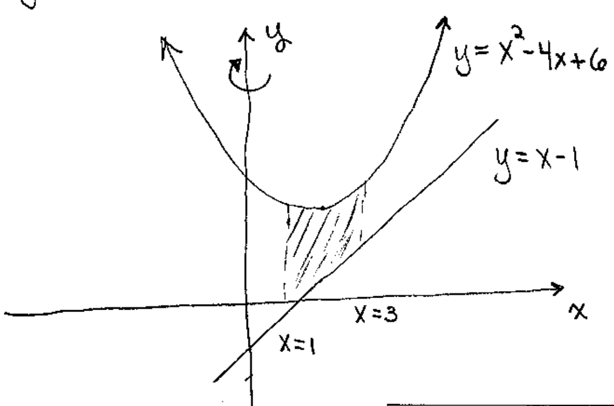
1. The region bounded by the graphs of $y = x^2 - 4x + 6$, $y = x - 1$, $x = 1$, and $x = 3$ is rotated about the $y$-axis to form a solid. Set up the definite integral that gives the volume of the solid. Then use your calculator (or computer) to evaluate the integral.

$y = x^2 - 4x + 6 = (x-2)^2 + 2$




Shells...

$$\text{Volume} = 2\pi \int_1^3 x\left(x^2 - 4x + 6 - x + 1\right) dx$$

$$= 2\pi \int_1^3 x\left(x^2 - 5x + 7\right) dx$$

$$\boxed{2\pi \int_1^3 \left(x^3 - 5x^2 + 7x\right) dx = \frac{28\pi}{3} \approx 29.3215}$$

2. Use the trapezoid rule with $n = 4$ to approximate $\int_1^3 e^{-x^2}\, dx$. Round your final answer to 4 digits. (Use enough decimal digits in your work so that your final answer has four correct digits.)

$h = \frac{3-1}{4} = 0.5 \Rightarrow x_0 = 1,\ x_1 = 1.5,\ x_2 = 2,\ x_3 = 2.5,\ x_4 = 3$

$f(x) = e^{-x^2}$

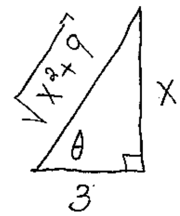
Trap rule estimate =

$$\frac{0.5}{2}\left[e^{-1} + 2e^{-2.25} + 2e^{-4} + 2e^{-6.25} + e^{-9}\right]$$

$\approx 0.1548234.$

$$\boxed{0.1548}$$

3. Use a trigonometric substitution to evaluate $\int \frac{5\,dx}{(x^2+9)^{3/2}}$.




$$x = 3\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$dx = 3\sec^2\theta \, d\theta$$

$$\int \frac{15\sec^2\theta\, d\theta}{(9\tan^2\theta+9)^{3/2}} = \int \frac{15\sec^2\theta\, d\theta}{(9\sec^2\theta)^{3/2}} = \frac{15}{27}\int \frac{1}{\sec\theta}\, d\theta = \frac{5}{9}\int \cos\theta\, d\theta$$

$$(9\sec^2\theta)^{3/2} = 27\sec^3\theta$$

$$= \frac{5}{9}\sin\theta + C$$

$$= \frac{5}{9}\,\frac{x}{\sqrt{x^2+9}} + C$$

$$\boxed{\frac{5x}{9\sqrt{x^2+9}} + C}$$

4. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n n^2}$.

RATIO TEST: $\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{3^{n+1}(n+1)^2} \cdot \frac{3^n n^2}{(x-2)^n} \right| = \lim_{n\to\infty} \frac{|x-2|}{3}\left(\frac{n}{n+1}\right)^2$

$$= \frac{|x-2|}{3} \cdot \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2 = \frac{|x-2|}{3}$$

ABS. CON. WHEN $\frac{|x-2|}{3} < 1 \Rightarrow |x-2| < 3$

$\Rightarrow (-1, 5)$

CHECK ENDPTS...

$x=-1$: $\sum \frac{(-3)^n}{3^n n^2} = \sum \frac{(-1)^n}{n^2}$ ABS. CON.

$p=2$

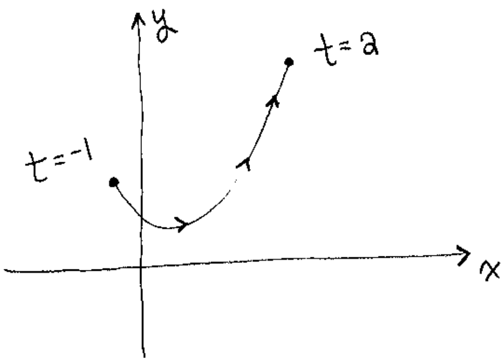
$x=5$: $\sum \frac{3^n}{3^n n^2} = \sum \frac{1}{n^2}$ ABS CON.

$$\boxed{\text{SERIES CONVERGES ABSOLUTELY ON } [-1, 5].}$$

7. Consider the curve described by the parametric equations

$$x = 2t + 1, \quad y = 3t^2 + 1, \qquad -1 \le t \le 2.$$

Find the area of the region between the curve and the $x$-axis.




$$A = \int_{-1}^{2} y(t)\, x'(t)\, dt$$

$$= \int_{-1}^{2} (3t^2+1)(2)\, dt$$

$$2t^3 + 2t \Big|_{-1}^{2} = (20) - (-4)$$

$$= 24$$

$$\boxed{24}$$

8. (a) Convert the point $(r, \theta) = \left(7, \dfrac{5\pi}{4}\right)$ to rectangular coordinates.

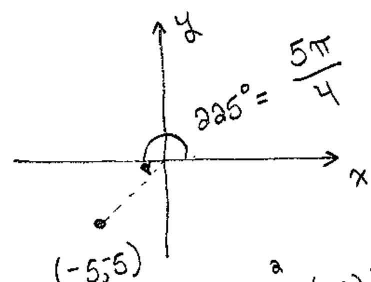
(b) Convert the point $(x, y) = (-5, -5)$ to polar coordinates.

(a)

$$x = 7\cos\frac{5\pi}{4} = 7\left(-\frac{\sqrt{2}}{2}\right) = \frac{-7\sqrt{2}}{2}$$

$$y = 7\sin\frac{5\pi}{4} = 7\left(-\frac{\sqrt{2}}{2}\right) = \frac{-7\sqrt{2}}{2}$$

(b)




$$r^2 = (-5)^2 + (-5)^2$$
$$= 50$$

$$r = 5\sqrt{2}$$

(a) $\left(-\dfrac{7\sqrt{2}}{2}, -\dfrac{7\sqrt{2}}{2}\right)$ (b) $\left(5\sqrt{2}, \dfrac{5\pi}{4}\right)$