Guidelines for Partial Fractions

To find a partial fraction decomposition (PFD), we choose the form of the PFD and then solve for undetermined coefficients. These guidelines will help:

1. If $P(x)/Q(x)$ is improper, do long division first. Then find the PFD of the fractional part.

2. Without loss of generality, we assume that $P(x)/Q(x)$ is proper. Completely factor $Q(x)$ into powers of linear factors and powers of irreducible quadratic factors.

3. For each linear factor of the form $(px+q)^m$, your PFD should include

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m},$$

   where $A_1, A_2, \ldots, A_m$ are constants to be determined.

4. For each irreducible quadratic factor of the form $(ax^2+bx+c)^n$, your PFD should include

$$\frac{C_1x+D_1}{(ax^2+bx+c)} + \frac{C_2x+D_2}{(ax^2+bx+c)^2} + \cdots + \frac{C_nx+D_n}{(ax^2+bx+c)^n},$$

   where the C's and D's are constants to be determined.

5. Write the appropriate form.

6. Solve for the undetermined coefficients. (There are many strategies!)