Math 151 - Test 1

February 24, 2016

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Determine a formula for the linear function whose graph passes through the points $(1, 7)$ and $(-3, 8)$. Write your answer in the form $f(x) = mx + b$.

$$\text{Slope} = m = \frac{8-7}{-3-1} = \frac{1}{-4} = -\frac{1}{4}$$

$$y = -\frac{1}{4}x + b$$

Use $(1,7)$ to get

$$7 = -\frac{1}{4}(1) + b \Rightarrow b = 7 + \frac{1}{4} = \frac{29}{4}$$

$$\boxed{f(x) = -\frac{1}{4}x + \frac{29}{4}}$$

2. (3 points) What is the domain of the function $g(x) = 5 + \sqrt{x+6}$? Write your answer in interval notation.

Must have $x + 6 \geq 0$

or

$x \geq -6 \Rightarrow$ $\boxed{\text{Domain is } [-6, \infty)}$

3. (3 points) Choose one of these relations that is NOT a function. Circle it, and then explain why it is not a function.

(A) $\{(1, 2), (2, 7), (3, 4), (4, -1)\}$

(B) $\{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$

(C) $\{(1, -1), (2, -2), (1, -1), (2, -2)\}$

(D) $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$ ← This is the only one that is not a function.

A single input (namely 1) corresponds to some different outputs (1, 2, 3, & 4).

4. (5 points) Write the relation $x^2 + y = 3 - 4x^2 + 2y$ as a function of x. Give your result in functional form. Then state the domain.

$$x^2 + y = 3 - 4x^2 + 2y$$

$$-3 + 5x^2 = y \longrightarrow \boxed{f(x) = 5x^2 - 3 \quad \text{Domain is } (-\infty, \infty)}$$

5. (6 points) Carefully describe how the graph of $h(x) = 2 + 3\sqrt{x-7}$ can be obtained from the graph of $f(x) = \sqrt{x}$.

① SHIFT RIGHT 7 UNITS. $(y = \sqrt{x-7})$

② VERTICALLY STRETCH BY FACTOR OF 3 $(y = 3\sqrt{x-7})$

③ SHIFT UP 2 UNITS $(y = 2 + 3\sqrt{x-7})$

6. (3 points) Consider the function defined by the equation $y = -17x + 9$. What are the domain and range of this function?

LINEAR FUNCTION

WITH NON ZERO SLOPE

DOMAIN = RANGE = $(-\infty, \infty)$

7. (5 points) Let $f(x) = 2x^2 + 3x$. Compute and simplify $f(x+a) - f(a)$.

$$\left[2(x+a)^2 + 3(x+a)\right] - \left[2a^2 + 3a\right]$$

$$= 2x^2 + 4xa + \cancel{2a^2} + 3x + \cancel{3a} - \cancel{2a^2} - \cancel{3a}$$

$$= \boxed{2x^2 + 4xa + 3x}$$

8. (10 points) Let $f(x) = x^2 + 2x - 3$. Determine the graph's x- and y-intercepts, the vertex, and two other points on the graph. Then carefully sketch the graph and determine the range of f. (Label your axes.)

X-INTS: $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3,\ x = 1$

$\boxed{(-3,0),\ (1,0)}$

Y-INT: $f(0) = -3$ $\boxed{(0,-3)}$

VERTEX: $x = \frac{-2}{2} = -1$

$f(-1) = (-1)^2 + 2(-1) - 3 = -4$

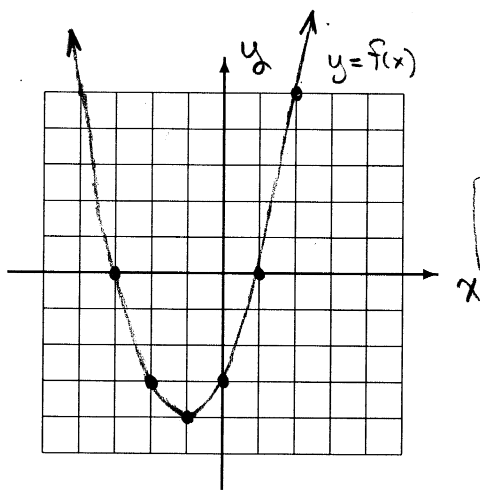
$\boxed{(-1,-4)}$

TWO OTHER PTS:

$f(2) = (2)^2 + 2(2) - 3 = 5$

$f(-2) = (-2)^2 + 2(-2) - 3 = -3$

$(2,5),\ (-2,-3)$



$\boxed{\text{RANGE IS } [-4,\infty)}$

9. (6 points) The total cost of manufacturing a set of golf clubs is given by

$$C(x) = 800 - 10x + 0.20x^2,$$

where x is the number of sets of golf clubs produced. How many sets of golf clubs should be manufactured to incur minimum cost and what is that minimum cost?

GRAPH OPENS UP. MIN OCCURS AT VERTEX: $x = \frac{10}{2(0.20)} = 25$

$C(25) = 800 - 10(25) + 0.20(25)^2$

$= 675$

MIN OF \$675 AT 25 SETS

10. (8 points) Consider the function $g(x) = -(x+3)^2 - 5$.

(a) What is the name we give to the graph of this function?

PARABOLA

(b) Determine the vertex of the graph.

SINCE THE FUNCTION IS WRITTEN IN VERTEX FORM, WE CAN READ THE VERTEX: $(-3,-5)$

(c) Does the graph open upward, downward, or neither? Explain how you know.

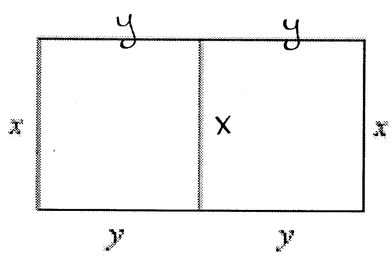
DOWNWARD. THE LEADING COEFFICIENT IS NEGATIVE.

(d) What are the domain and range of g?

DOMAIN: $(-\infty,\infty)$

RANGE: $(-\infty,-5]$

11. (10 points) Cindy wants to construct two side-by-side dog-training pens as shown below. She has 400 ft of fencing material to use. What values of x and y maximize the combined areas of the pens? (You must show all work for full credit.)



$3x + 4y = 400$

$4y = 400 - 3x$

$y = \dfrac{400-3x}{4}$

MAXIMIZE $A = xy$

$A = x\left(\dfrac{400-3x}{4}\right)$

$= \dfrac{400x - 3x^2}{4}$

VERTEX AT $X = \dfrac{-400/4}{2(-3/4)}$

$= 66.\overline{6}$

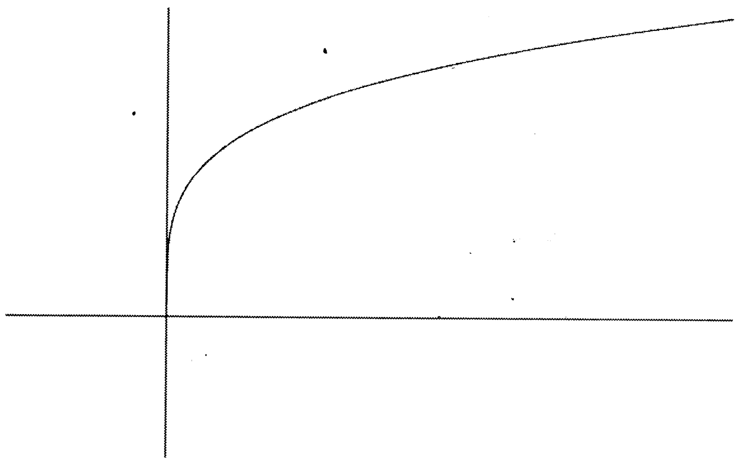
$y = \dfrac{400 - 3(66.\overline{6})}{4}$

$= 50$

$\boxed{\begin{array}{l} X = 66.6 \text{ FT} \\ y = 50 \text{ FT} \end{array}}$

12. (4 points) The graph of a function is shown below. Even though no scale is shown, you should be able to draw some conclusions about the function. Which of these could **not** possibly be the function? Circle all that apply.

$y = 5\sqrt{x}$ $\quad$ $y = x^5$ $\quad$ $y = \sqrt[4]{x}$ $\quad$ $y = -2\sqrt{x}$ $\quad$ $y = \frac{1}{2}\sqrt[4]{x}$ $\quad$ $y = \frac{2}{x^3}$ $\quad$ $y = \sqrt[3]{x}$



13. (4 points) The graph of $f(x) = \frac{1}{x^3}$ is shifted 2 units right and 3 units down to create the graph of a new function. What is that new function?

$$\boxed{g(x) = \frac{1}{(x-2)^3} - 3}$$

2 RIGHT

3 DOWN

14. (2 points) Explain how you can tell that the quadratic function $g(x) = -5x^2 - 100x + 67$ has no minimum value.

BECAUSE THE LEADING COEFF IS NEG, THE GRAPH OF THIS QUAD. FUNC. IS A DOWNWARD OPENING PARABOLA.

THE VERTEX IS AT THE HIGH POINT, BUT THERE IS NO LOWEST POINT ON GRAPH.

15. (8 points) Consider the function

$$f(x) = \begin{cases} x^2+1, & x < -5 \\ |x+6|+2, & -5 < x < 0 \\ \sqrt{2x}, & x > 0 \end{cases}$$

(a) Evaluate $f(8)$.

$f(8) = \sqrt{2(8)} = \sqrt{16} = \boxed{4}$

(b) Evaluate $f(0)$.

$f(0)$ IS NOT DEFINED.

(c) Evaluate $f(-10)$

$f(-10) = (-10)^2 + 1 = 100 + 1 = \boxed{101}$

(d) What is the domain of f?

$X \neq -5$

$X \neq 0$ $\Longrightarrow$ $\boxed{(-\infty,-5) \cup (-5,0) \cup (0,\infty)}$

16. (2 points) The graph of $y = -3(x-4)^5 + 7$ is a transformed version of the graph of what basic function?

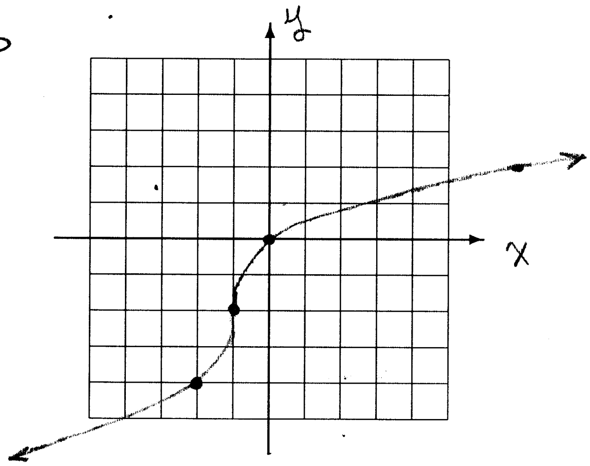
$\boxed{y = x^5}$

17. (8 points) **Carefully sketch** the graph of each function. Your graph should show details such as correct scale and position. (Label your axes.)

(a) $g(x) = 2\sqrt[3]{x+1} - 2$

STRETCHED AND SHIFTED $y = \sqrt[3]{x}$

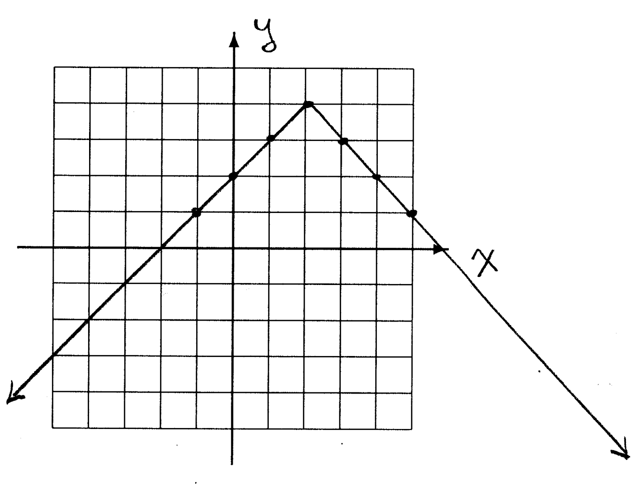
X	g(x)
-1	-2
0	0
-2	-4
7	2



(b) $f(x) = 4 - |x - 2|$

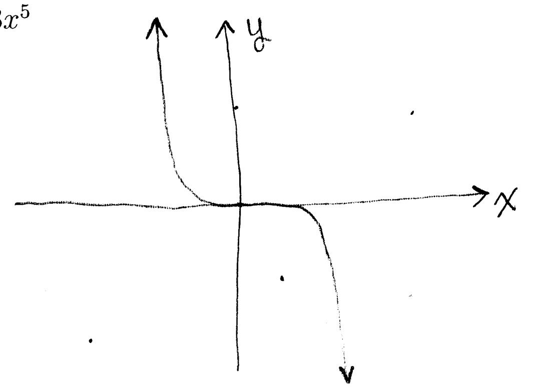
FLIPPED AND SHIFTED $y = |x|$

x	f(x)
5	1
4	2
3	3
2	4
1	3
0	2
-1	1

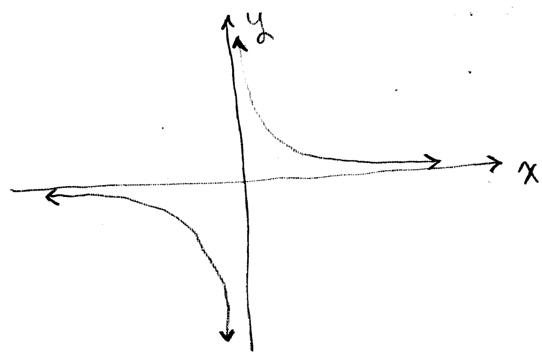


18. (6 points) Very roughly, sketch the general shape of the graph of each function.

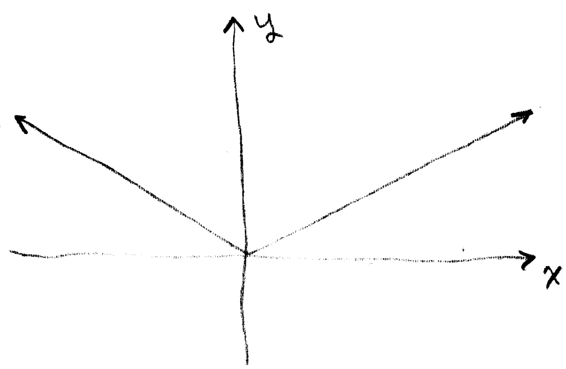
(a) $h(x) = -3x^5$



(b) $f(x) = \dfrac{1}{2x}$



(c) $g(x) = \dfrac{|x|}{3}$



19. (2 points) Referring back to part(c) of the problem above, explain the difference between the graphs of $f(x) = 3|x|$ and $g(x) = \dfrac{|x|}{3}$.

THE GRAPH OF f(x) IS A VERTICALLY STRETCHED VERSION OF THE GRAPH OF g(x).