Math 151 - Test 2
March 16, 2016

Name key

Score

Show all work. Supply explanations where necessary.

1. (6 points) Let $f(x) = 3x^2 - 8x + 1$ and $g(x) = x^2 + 8x - \sqrt{x}$.

   (a) Find and simplify a formula for $(f+g)(x)$.

   $(f+g)(x) = 3x^2 - 8x + 1 + x^2 + 8x - \sqrt{x} = \boxed{4x^2 + 1 - \sqrt{x}}$

   (b) Evaluate $(f+g)(4)$.

   $(f+g)(4) = 4(4)^2 + 1 - \sqrt{4} = \boxed{63}$

   (c) Determine the domain of $f+g$.

   $[0, \infty)$

2. (8 points) Some values of the functions f and g are given in the table below. Use the data from the table to evaluate each of the following.

x	1	2	3	4	5
$f(x)$	-1	3	-2	5	0
$g(x)$	7	0	4	-7	-2

   (a) $(fg)(3)$

   $= f(3)g(3) = (-2)(4) = \boxed{-8}$

   (b) $\left(\frac{g}{f}\right)(5) = \frac{g(5)}{f(5)} = \frac{-2}{0}$ $\boxed{\text{NOT DEFINED}}$

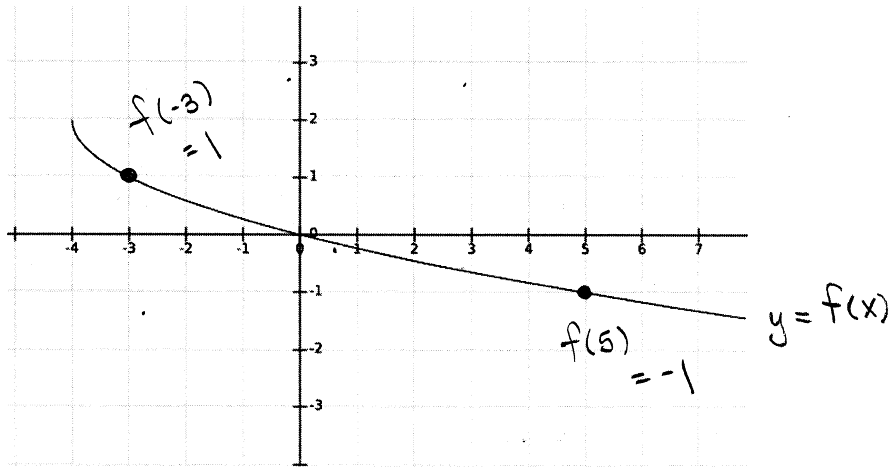
   (c) $(f+f)(4)$

   $f(4) + f(4) = 5 + 5 = \boxed{10}$

   (d) $(f \circ g)(3)$

   $f(g(3)) = f(4) = \boxed{5}$

3. (4 points) Let $g(x) = 3x - 5$. Using the function g and the graph of $y = f(x)$ shown below, compute each of the following.



(a) $(g - f)(5)$ $= g(5) - f(5) = [15-5] - (-1) = \boxed{11}$

(b) $(f + g)(-3)$ $= f(-3) + g(-3) = 1 + [-9-5] = \boxed{-13}$

4. (6 points) Let $f(x) = x^2 + 3x - 5$ and $g(x) = x - 4$.

(a) Find and simplify the formula for $(f \circ g)(x)$.

$$f(g(x)) = f(x-4) = (x-4)^2 + 3(x-4) - 5 = x^2 - 8x + 16 + 3x - 12 - 5$$
$$= \boxed{x^2 - 5x - 1}$$

(b) What is the domain of $(f \circ g)$?

All real #'s : $\boxed{\mathbb{R}}$

5. (4 points) Find two functions f and g so that $(f \circ g)(x) = \sqrt[5]{2x^3 + 7}$.

$$\boxed{\begin{array}{l} g(x) = 2x^3 + 7 \\ f(x) = \sqrt[5]{x} \end{array}}$$

6. (4 points) Let $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$. Evaluate each of the following.

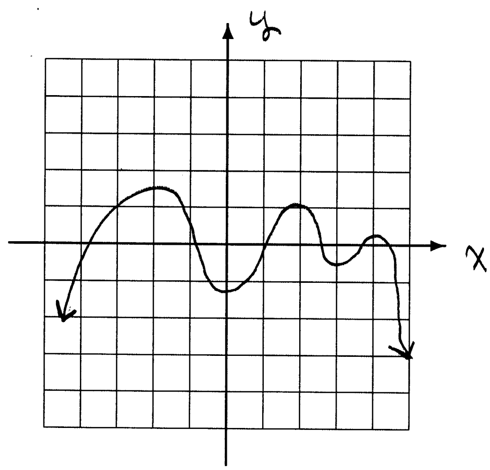
(a) $(f \circ g)(4)$

$f(g(4)) = f(9) = \boxed{3}$

(b) $(g \circ f)(9)$

$g(f(9)) = g(3) = \boxed{7}$

7. (4 points) Sketch the graph of a function that is NOT one-to-one. Explain why it is not.



THE GRAPH FAILS THE HORIZONTAL LINE TEST... SOME y-VALUES ARE ASSOCIATED WITH MULTIPLE X-VALUES.

8. (8 points) Find the inverse of $g(x) = 1 + \sqrt{x-2}$. Be sure to state the domain of g^{-1}.

$y = 1 + \sqrt{x-2}$

$y - 1 = \sqrt{x-2}$

$(y-1)^2 = x - 2$

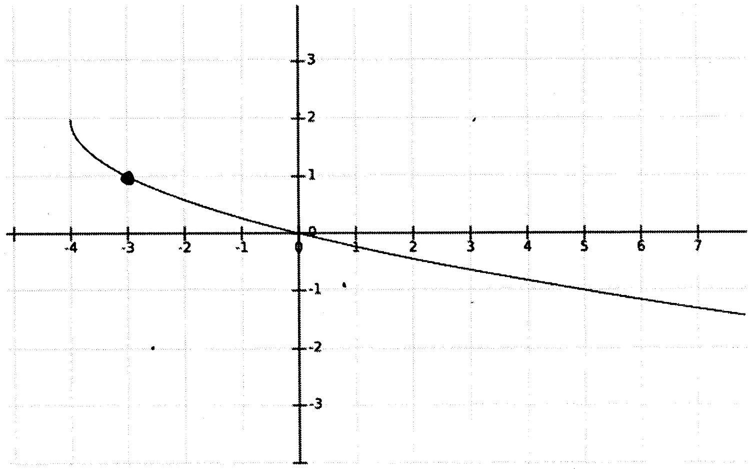
$(y-1)^2 + 2 = x$

$g^{-1}(x) = (x-1)^2 + 2$

DOMAIN OF g^{-1} = RANGE OF g

$= [1, \infty)$

9. (2 points) The graph of $f(x)$ is shown below. Use the graph to determine $f^{-1}(1)$.



$f^{-1}(1) = y$

$\Updownarrow$

$f(y) = 1$

$\Downarrow$

$y = \boxed{-3}$

10. (6 points) Use compositions to show that $f(x) = 6 - \sqrt[3]{x}$ and $g(x) = (6-x)^3$ are inverses.

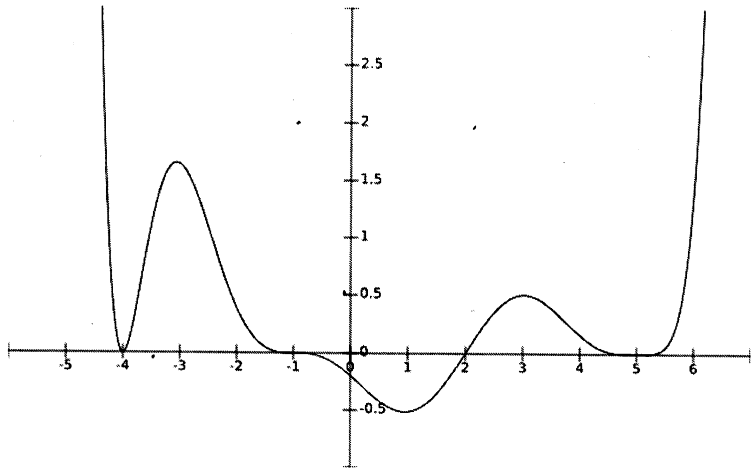
$$f(g(x)) = 6 - \sqrt[3]{(6-x)^3}$$
$$= 6 - (6-x)$$
$$= 6 - 6 + x$$
$$= x \checkmark$$

$$g(f(x)) = (6 - (6 - \sqrt[3]{x}))^3$$
$$= (6 - 6 + \sqrt[3]{x})^3$$
$$= (\sqrt[3]{x})^3 = x \checkmark$$

11. (4 points) Let $f(x) = (x-2)^2 - 7$. Find the domain of f^{-1}.

Domain of f^{-1} = Range of f = $\boxed{[-7, \infty)}$

12. (10 points) The graph of a polynomial is shown below.



(a) Is the degree even or odd?

EVEN -- SAME END BEHAVIOR ON BOTH ENDS

(b) Is the leading coefficient positive or negative?

POSITIVE -- OPENS UP

(c) Which zeros have multiplicity one?

$X = 2$ (STRAIGHT THROUGH AXIS)

(d) Which zeros have even multiplicity?

$X = -4$, $X = 5$ (BOUNCES)

(e) Which zeros have odd multiplicity greater than 1?

$X = -1$ (FLATTENS AND CROSSES)

13. (12 points) Consider the polynomial $f(x) = -x(x-2)^3(2x+1)^2$.

(a) Determine the degree of f and the leading coefficient.

DEGREE:

$1 + 3 + 2 = \boxed{6}$

LEADING COEFF:

$-1(1)^3(2)^2 = \boxed{-4}$

(b) State the zeros of f and their corresponding multiplicities.

$X = 0$ MULT 1

$X = 2$ MULT 3

$X = -\frac{1}{2}$ MULT 2

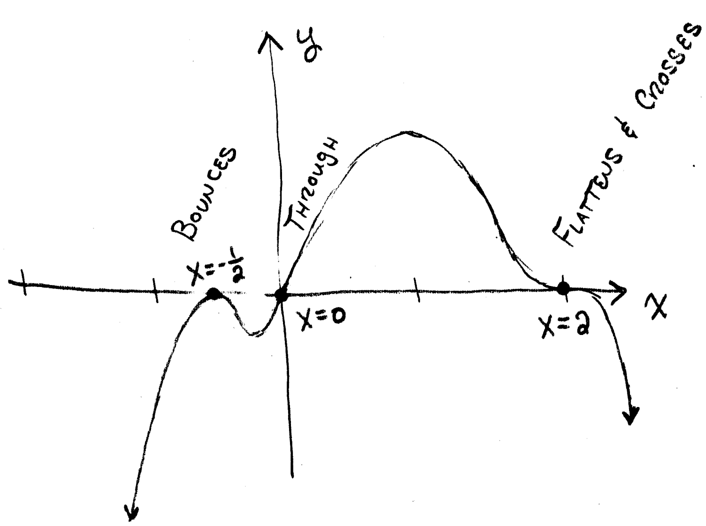
(c) Describe the end behavior of the graph of f. (A picture or diagram will work!)

$-4x^6 \Longrightarrow$

(d) Determine the y-intercept.

$f(0) = 0 \Rightarrow$ y-INT IS $(0,0)$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior near the x-intercepts.

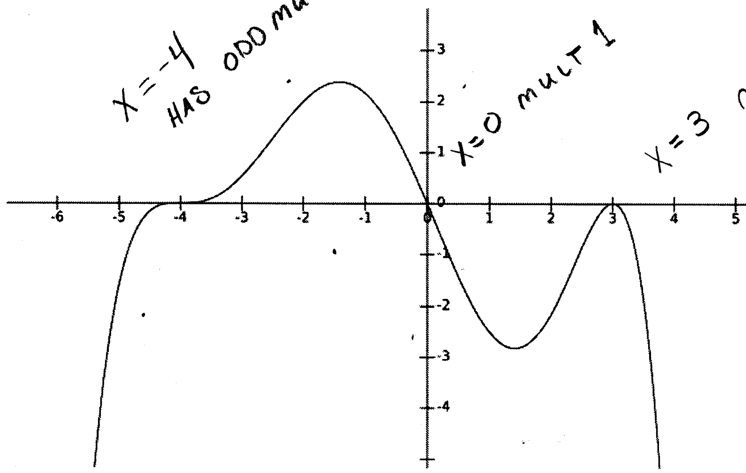


14. (8 points) Give the factored form of a polynomial whose graph has the same general shape of the one given below.

X = -4 HAS ODD MULT 7\

X = 0 MULT 1

X = 3 MULT EVEN



DEGREE EVEN

NEG. LEADING COEFF.

$$\boxed{f(x) = -(x+4)^3(x)(x-3)^2}$$

15. (4 points) Let $p(x)$ be the polynomial whose graph is shown above. Use the graph to solve the inequality $p(x) \geq 0$.

ABOVE OR ON X-AXIS

$$\boxed{[-4,0] \cup [3,3]}$$

OR $-4 \leq x \leq 0$ OR $x = 3$

16. (10 points) Find the exact values of the real and complex zeros of $p(x) = x^3 - 4x^2 + 5x$. Show all work.

$$x(x^2-4x+5) = 0$$

$$x = 0 \quad \text{or} \quad x^2 - 4x + 5 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2} = \frac{4 \pm \sqrt{-4}}{2} = 2 \pm i$$

$$\boxed{x = 0, \; x = 2+i, \; x = 2-i}$$