Math 151 - Test 3
April 20, 2016

Name key

Score _____

Show all work. You will not receive credit if work is not shown. Supply explanations where necessary.

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1. (8 points) Construct a 3rd degree polynomial with real coefficients such that two of its zeros are 0 and $2+i$ and its leading coefficient is 4. Write your final answer in standard form (not factored form).

Zeros are $x=0$, $x=2+i$, $x=2-i$

Poly must be $4x(x-2-i)(x-2+i)$

$= 4x(x^2 - 2x + ix - 2x + 4 - 2i - ix + 2i - i^2)$

$= 4x(x^2 - 4x + 5)$

$= \boxed{4x^3 - 16x^2 + 20x}$

2. (4 points) Find the degree and the y-intercept: $f(x) = (x+1)(x-2)^7$

Degree is $1+7 = \boxed{8}$

y-int: $x=0 \Rightarrow y = f(0) = (1)(-2)^7 = -128$

$\boxed{(0, -128)}$

3. (12 points) Consider the polynomial $p(x) = x^4 + x^3 - 9x^2 - 7x + 14$. Find the complete factorization of p. **You will only be given credit for the work you show.** (Hint: Two rational zeros should be easy to find. Use synthetic division to deflate.)

$14: \pm\{1, 2, 7, 14\}$

$1: \pm\{1\}$

$\Rightarrow$ POSSIBLE RAT ZEROS ARE $\pm\{1, 2, 7, 14\}$

A QUICK GLANCE AT THE GRAPH RULES OUT ALL BUT $\{-2, 1\}$

SEE NEXT PAGE

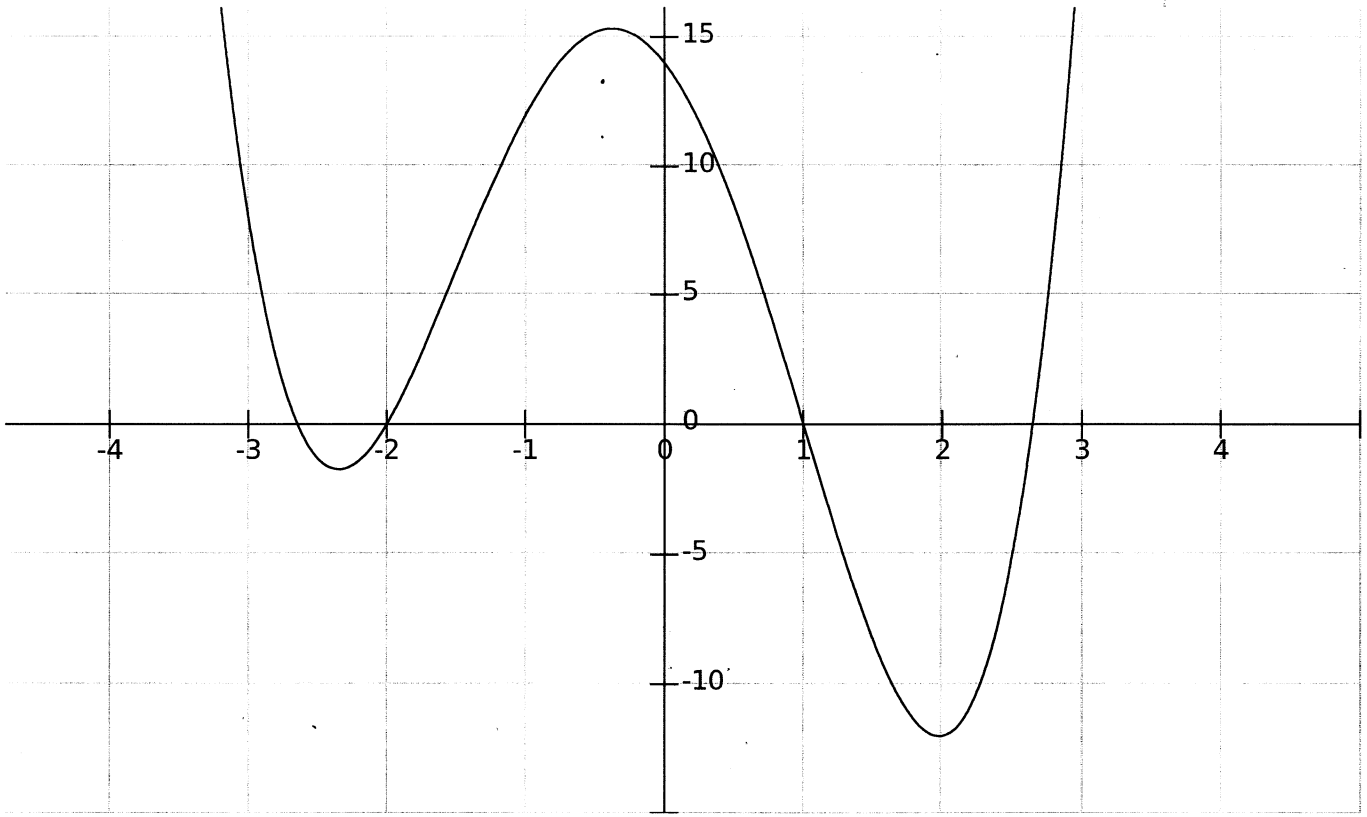
$$\begin{array}{r|rrrr|r} -2 & 1 & 1 & -9 & -7 & 14 \\ & & -2 & 2 & 14 & -14 \\ \hline 1 & 1 & -1 & -7 & 7 & 0 \\ & & 1 & 0 & -7 & \\ \hline & 1 & 0 & -7 & 0 & \end{array}$$

$(x+2)(x-1)(x^2-7)$

$(x-\sqrt{7})(x+\sqrt{7})$

FINAL ANSWER:

$$(x+2)(x-1)(x-\sqrt{7})(x+\sqrt{7})$$


15
10
5
0
-5
-10
-4
-3
-2
-1
0
1
2
3
4

4. (4 points) A polynomial with only real coefficients has the zeros $2+3i$ and $-9i$. What are another two of its zeros?

Conjugates: $2-3i$ & $9i$

5. (8 points) Find the real and complex zeros of $g(x) = x^3 - 10x^2 + 34x$.

Zeros are $x = 0$, $x = 5+3i$, $x = 5-3i$

$x(x^2 - 10x + 34)$

$x = \frac{10 \pm \sqrt{100 - 4(34)}}{2}$

$= \frac{10 \pm \sqrt{-36}}{2} = \frac{10 \pm 6i}{2}$

$= 5 \pm 3i$

6. (4 points) Use the Rational Zeros Theorem to list all possible rational zeros.

$$p(x) = 3x^4 - 5x^3 + 5x - 2$$

$-2: \pm\{1, 2\}$

$3: \pm\{1, 3\}$

Possible rat zeros are $\pm\{1, \frac{1}{3}, 2, \frac{2}{3}\}$

7. (8 points) Completely factor: $x^3 - 2x^2 - x + 2$

$x^2(x-2) - 1(x-2)$

$(x-2)(x^2-1) = (x-2)(x-1)(x+1)$

8. (6 points) Use synthetic division to write the expression below in the form $q(x)+\frac{r(x)}{d(x)}$.

$$\frac{x^3+5x^2-17x-13}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -17 & -13 \\ & & 2 & 14 & -6 \\ \hline & 1 & 7 & -3 & -19 \end{array}$$

$$= x^2+7x-3-\frac{19}{x-2}$$

9. (6 points) Use synthetic division to show that $(x-3)$ is a factor of $x^4-3x^3+7x-21$.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 0 & 7 & -21 \\ & & 3 & 0 & 0 & 21 \\ \hline & 1 & 0 & 0 & 7 & 0 \end{array}$$

REMAINDER IS ZERO!

10. (8 points) Use long division to write the expression below in the form $q(x)+\frac{r(x)}{d(x)}$.

$$\frac{x^3+2x^2+3x-7}{x^2+x-2}$$

$$\begin{array}{r} x+1 \\ x^2+x-2 \overline{)\, x^3+2x^2+3x-7} \\ \underline{x^3+x^2-2x} \\ x^2+5x-7 \\ \underline{x^2+x-2} \\ 4x-5 \end{array}$$

$$= x+1+\frac{4x-5}{x^2+x-2}$$

11. (12 points) Consider the polynomial $f(x) = -(x+1)^3(x-4)(3x-1)^2$.

(a) Determine the degree of f and the leading coefficient.

Degree: $3+1+2 = 6$

Leading C: $(-1)(1)^3(1)(3)^2 = -9$

(b) State the zeros of f and their corresponding multiplicities.

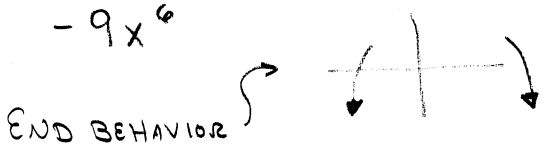
$x = -1$ mult 3

$x = 4$ mult 1

$x = \frac{1}{3}$ mult 2

(c) Describe the end behavior of the graph of f. (A picture or diagram will work!)

Leading term is $-9x^6$

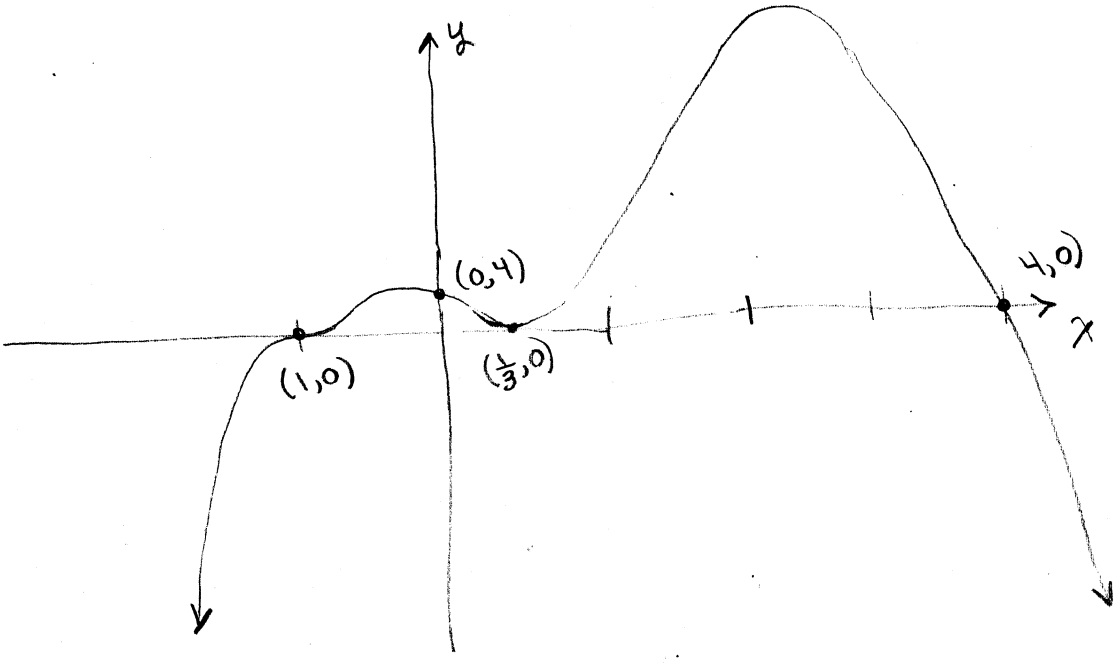
End behavior 

(d) Determine the y-intercept.

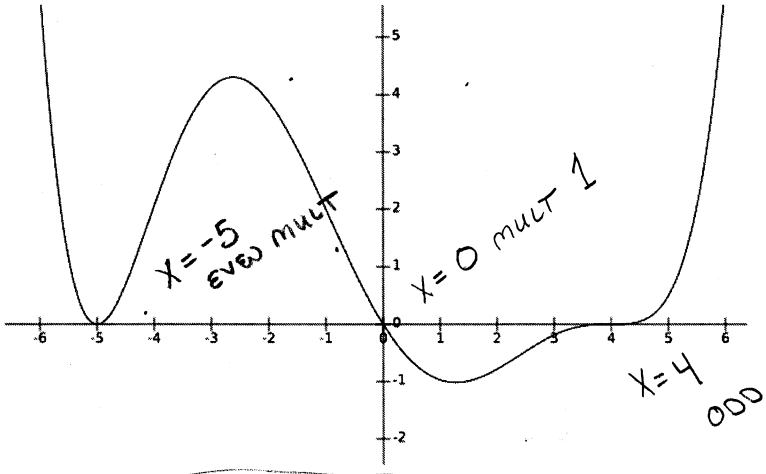
$x = 0 \Rightarrow y = f(0) = -(1)^3(-4)(-1)^2 = 4$

$(0,4)$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior near the x-intercepts.



12. (6 points) Give the factored form of a polynomial whose graph has the same general shape of the one given below.



GRAPH OPENS UP ⇓ LEADING COEFF +

$x(x+5)^2(x-4)^3$

13. (6 points) Determine the vertical asymptotes of the graph of $R(x) = \dfrac{x+1}{x^2-5x+6}$.

DENOM = 0, NUMER ≠ 0 $(x-2)(x-3)$

AT $x=2$, $x=3$

V.A. ARE $x=2$, $x=3$

14. (4 points) What is the domain of the rational function $F(x) = \dfrac{x-5}{x+5}$?

$x+5 \neq 0 \Rightarrow x \neq -5$

OR

$(-\infty, -5) \cup (-5, \infty)$

15. (4 points) What are the x- and y-intercepts of the graph of the function $F(x)$ in problem #14?

X-INTS: $y=0 \Rightarrow x-5=0 \Rightarrow x=5$ $(5,0)$

y-INT: $x=0 \Rightarrow y=F(0)=\dfrac{-5}{5}=-1$ $(0,-1)$