Math 153 - Test 2
October 9, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) The data below show the maximum daily temperatures (in °F) at Chicago's O'Hare Airport during the month of April 2014.

38	43	45	48	51	51	51	52	53	54
55	57	57	57	59	60	60	61	62	62
63	64	66	66	69	70	70	74	79	80

(a) Find the percentile for 66°F.

$$\frac{\#\text{ OF VALUES } < 66}{30} = \frac{22}{30} = 0.7\overline{3}$$

73.33%

$\rightarrow$ 73RD PERCENTILE

(b) Find the temperature at the 35th percentile.

$$\frac{L}{30} = 0.35 \Rightarrow L = 10.5 \Rightarrow L = 11$$

11TH VALUE IS $\boxed{55°}$

(c) Find the temperature at the 60th percentile.

$$\frac{L}{30} = 0.60 \Rightarrow L = 18$$

$$\frac{18^{TH} + 19^{TH}}{2} = \frac{61 + 62}{2} = \boxed{61.5°}$$

2. (18 points) Believe it or not, *zenzizenizenic* is an actual word. Suppose a letter is selected at random from this word.

(a) Give a possible sample space for this experiment.

$\{z, e, n, i, c\}$

(b) Is your sample space uniform? Explain.

No, each outcome is not equally likely as there are 4 z's, 3 e's, 3 n's, 3 i's, and 1 c.

(c) Pick two outcomes from your sample space and give their probabilities.

$P(\{z\}) = \frac{4}{14}$

$P(\{c\}) = \frac{1}{14}$

(d) Are the probabilities given above theoretical, experimental, or subjective? Explain.

The probs were based on assuming each of the 14 letters is equally likely and counting.

(e) State an event that has probability less than 0.5, and give the probability of your event.

A = event of selecting a vowel

$= \{e, i\}$ $\quad P(A) = \frac{6}{14} \approx 42.86\%$

(f) Steve claims that the odds of selecting the letter z are 4/14. Is Steve correct? Explain.

Steve is wrong. His number is a probability, not odds. Odds of selecting z are

$\frac{4}{10} = \frac{2}{5}$

3. (3 points) After doing all her homework and studying very hard, Julie announced, "There is a 100% chance that I will ace this statistics test!" What type of probability (theoretical, experimental, subjective, or geometric) did Julie compute? Explain.

SUBJECTIVE --- JULIE BASED HER PROBABILITY ON HER KNOWLEDGE OF THE EXPERIMENT, BUT SHE DID NOT COUNT, MEASURE, OR EXPERIMENT.

4. (8 points) A card is drawn at random from a standard deck of playing cards. Let A be the event of drawing a face card, and let B be the event of drawing a queen. Compute $P(A|B)$ and $P(B|A)$. Be sure to indicate which is which. Are A and B independent?

$P(A|B)$ = PROB OF FACE CARD GIVEN QUEEN = $\boxed{100\%}$

$P(B|A)$ = PROB OF QUEEN GIVE FACE CARD = $\frac{4}{12} = \boxed{\frac{1}{3}}$

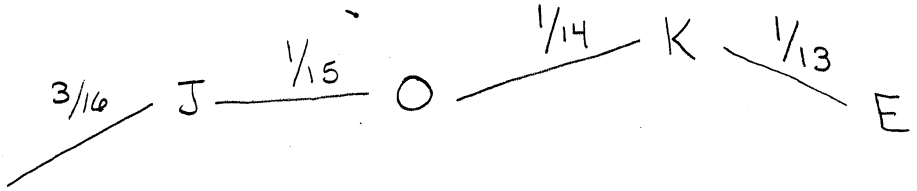
NOT INDEPENDENT SINCE $P(A) = \frac{12}{52} \neq P(A|B) = 100\%$

5. (4 points) The odds against winning a certain game are 13 to 2. What is the probability of winning?

ODDS AGAINST = $\frac{13}{2} \Rightarrow$ ODDS IN FAVOR = $\frac{2}{13}$

$\Rightarrow$ PROB OF WINNING = $\frac{2}{2+13} = \boxed{\frac{2}{15}}$

6. (8 points) Four letters are chosen at random (without replacement) from the word *EYJAFJALLAJOKULL*. What is the probability of spelling *JOKE* (in order)?



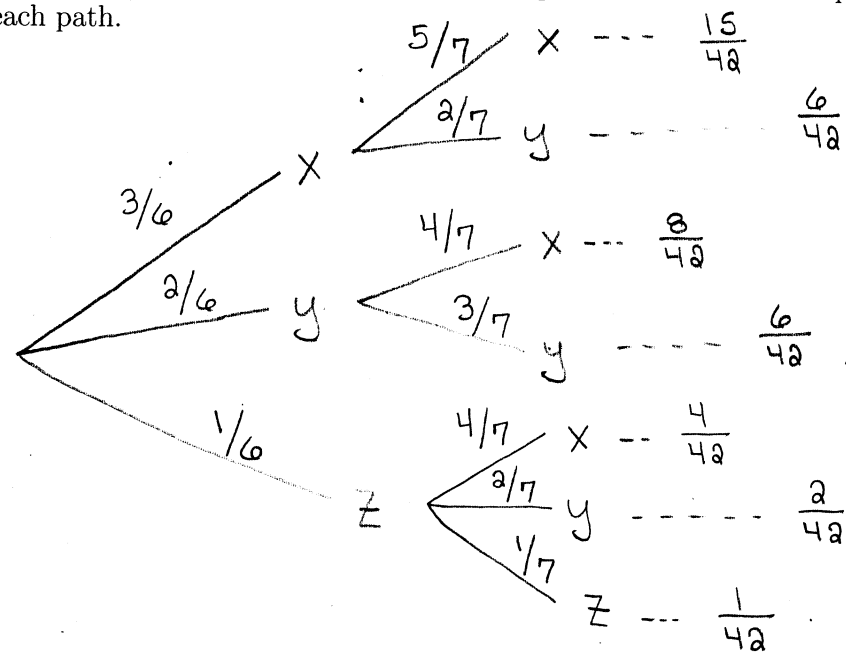
$\frac{3}{16} \times \frac{1}{15} \times \frac{1}{14} \times \frac{1}{13} = \boxed{\frac{3}{43680}}$

7. (12 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

x	x	x	y	y	z

x	x	x	x	y	y

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) What is the probability of drawing at least one letter x?

$$\underset{xx}{\frac{15}{42}} + \underset{xy}{\frac{6}{42}} + \underset{yx}{\frac{8}{42}} + \underset{zx}{\frac{4}{42}} = \boxed{\frac{33}{42}}$$

(c) What is the probability of drawing the letter z from the second box?

$$P(\{zz\}) = \boxed{\frac{1}{42}}$$

(d) What is the probability of drawing the letter y from the first box and the letter x from the second box?

$$P(\{yx\}) = \boxed{\frac{8}{42}}$$

8. (8 points) The Ebola virus was first identified in 1976. Since then, there have been outbreaks in 16 different years. The data below show the numbers of people infected by the Ebola virus in those 16 years, in the order in which they occurred. (The last number is current as of October 8, 2014.)

~~603~~ ~~1~~ ~~34~~ ~~53~~ ~~315~~ ~~99~~ ~~425~~ ~~122~~
~~143~~ ~~35~~ ~~17~~ ~~413~~ ~~32~~ ~~1~~ ~~53~~ ~~4461~~

Find the five-number summary, the interquartile range, and the cutoff values for outliers.

1, 1, 17, 32, 34, 35, 53, 53, 99, 122, 143, 315, 413, 425, 603, 4461

$Q_1 = 33$

MEDIAN $= \frac{53+99}{2} = 76$

$Q_3 = \frac{315+413}{2} = 364$

5-NUMBER SUMM:
MIN = 1, $Q_1 = 33$,
MED = 76, $Q_3 = 364$
MAX = 4461

IQR = 364 − 33 = 331

CUTOFFS:
$33 - 1.5(331) = -463.5$
$364 + 1.5(331) = 860.5$

9. (8 points) Since the identification of the virus, 6807 people have been infected with Ebola. Of those infected, the disease was fatal in 3885 cases.

(a) Estimate the probability that an infected person recovers from the Ebola virus. Write your answer as a percent, rounded to the nearest whole percent.

$1 - \frac{3885}{6807} \approx 43\%$

(b) What are the odds in favor of recovering from the Ebola virus?

$\frac{2922}{3885}$ OR APPROX $\frac{43}{57}$

(c) If you are certain that you will be infected by the Ebola virus in both of the next two outbreaks, what is the probability that you will survive both?

$(0.43)(0.43) = 0.1849$

$\approx 18\%$

10. (9 points) Percy's overall percentage in Calculus I was 81.4, while the class mean was 71.2 and the standard deviation of 5.6. His overall percentage in Calculus II was 82.1, while the class mean was 73.0 and the standard deviation was 7.4.

(a) Compute Percy's z-scores.

CALC I: $\frac{81.4 - 71.2}{5.6} \approx \boxed{1.82}$ CALC II: $\frac{82.1 - 73}{7.4} \approx \boxed{1.23}$

(b) Relatively speaking, in which class did Percy do better?

CALC I WHERE HE HAD THE GREATER Z-SCORE.

(c) Did he do unusually well in either class? Explain.

NO, NEITHER Z-SCORE IS GREATER THAN 2.

11. (10 points) Suppose A and B are events such that $P(A) = 0.52$, $P(\overline{B}) = 0.36$, and $P(A \cup B) = 0.68$.

(a) Compute $P(B)$.

$1 - 0.36 = \boxed{0.64}$

(b) Compute $P(A \cap B)$.

$P(A \cap B) = 0.52 + 0.64 - 0.68 = \boxed{0.48}$

(c) Compute $P(B|A)$.

$P(B|A) = \frac{0.48}{0.52} \approx \boxed{0.923}$

(d) Are A and B independent? Explain.

NO, $P(B|A) \neq P(B)$

(e) What are the odds against A?

$P(A) = 0.52 \Rightarrow$ ODDS IN FAVOR $= \frac{0.52}{0.48}$

$\Rightarrow$ ODDS AGAINST $= \frac{48}{52} = \boxed{\frac{12}{13}}$