Math 153 - Test 3a

November 13, 2014

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (3 points) Ages of registered U.S. automobiles are normally distributed with mean 96 months and standard deviation 16 months. In a random sample of 24 vehicles, what is the probability that the mean age is greater than 100 months?

$\mu_{\bar{x}} = 96$

$\sigma_{\bar{x}} = \frac{16}{\sqrt{24}}$

$P(\bar{x} > 100) = \text{normalcdf}(100, 99999, 96, \frac{16}{\sqrt{24}})$

$\approx \boxed{0.1103}$

2. (3 points) In a recent Gallup Poll survey of 1015 U.S. adults, 396 said they use text messaging very often. Find a 95% confidence interval estimate for the population proportion of adults who use text messaging very often.

1-PropZInt

x: 396

n: 1015

C-Level: 0.95

95% C.I. ESTIMATE IS (0.36014, 0.42016)

WE CAN BE 95% CONFIDENT THAT THE TRUE POP. PROPORTION IS BETWEEN 36% AND 42%.

3. (3 points) Sixty-six percent of U.S. adults are satisfied with how the American health-care system works for them. If 20 adults are selected at random, what would be an unusually small number of satisfied adults?

BINOMIAL

$p = 0.66$

$q = 0.34$

$N = 20$

$\mu = 20(0.66) = 13.2$

$\sigma = \sqrt{20(0.66)(0.34)} \approx 2.12$

$\mu - 2\sigma \approx 13.2 - 2(2.12) = 8.96$

AN UNUSUALLY SMALL NUMBER OF SATISFIED ADULTS IS 8 OR FEWER.

4. (3 points) Mortgage interest rates are normally distributed with mean 3.950 and standard deviation 0.102. Steve will not consider an interest rate offer from a bank unless it is in the lowest 30% of rates. What is the greatest interest rate that Steve will consider?

$$\text{invNorm}(0.30, 3.950, 0.102) \approx \boxed{3.90}$$

5. (3 points) Given the following probability distribution, identify the unusually small values of x. Explain your reasoning.

x	0	1	2	3	4	5	6	7
$P(x)$	0.012	0.026	0.020	0.001	0.472	0.002	0.324	0.143

$P(x \leq 0) = 0.012 < 0.05$

$P(x \leq 1) = 0.012 + 0.026 = 0.038 < 0.05$

$P(x \leq 2) = 0.012 + 0.026 + 0.020 = 0.058 > 0.05$

X = 0 & X = 1 ARE UNUSUALLY SMALL

6. (3 points) A federal report indicated that 27% of children had a good diet. How large a sample is needed to estimate the true proportion of children with good diets within 2% with a 95% confidence interval?

$1-\alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$z_{\alpha/2} \approx 1.96$

$\hat{p} = 0.27$

$\hat{q} = 0.73$

$E = 0.02$

$$N \approx \frac{(1.96)^2(0.27)(0.73)}{(0.02)^2} \approx 1892.95$$

CHOOSE N = 1893

7. (3 points) According to *Time* magazine, 37% of people believe that places could be haunted. In a random sample of 12 people, what is the probability that fewer than 4 believe in haunted places?

BINOMIAL

$p = 0.37$

$N = 12$

$$P(x<4) = P(x \le 3)$$

$$= \text{binomcdf}(12, 0.37, 3)$$

$$\approx \boxed{0.2947}$$

8. (4 points) Lengths of pregnancies are normally distributed with mean 268 days and standard deviation 15 days. In 200 pregnancies, about how many are shorter than 260 days or longer than 275 days?

$$P(x<260) + P(x>275)$$

$$= \text{normalcdf}(-99999, 260, 268, 15)$$

$$+ \text{normalcdf}(275, 99999, 268, 15)$$

$$= 0.61727\ldots \approx 61.73\%$$

61.73% OF 200 $\approx$ 123.45

$\boxed{\text{ABOUT 123 PREGNANCIES}}$

Math 153 - Test 3b

November 13, 2014

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (8 points) In a certain neighborhood, the probability distribution for the number of children per household, x, is summarized as follows.

x	$P(x)$
0	0.148
1	0.289
2	0.313
3	0.164
4	0.047
5	0.023
6	0.008
7	0.008

(a) What is the probability that a randomly selected household has 4 or more children?

$$P(x \geq 4) = 0.047 + 0.023 + 0.008 + 0.008 = \boxed{0.086}$$

(b) What is the mean number of children per household?

$$\mu = 0(0.148) + 1(0.289) + 2(0.313) + \cdots + 7(0.008) = \boxed{1.814}$$

(c) What is the standard deviation?

$$\sigma^2 = 0(0.148) + 1(0.289) + 4(0.313) + \cdots + 49(0.008) - (1.814)^2$$

$$= 1.733404 \Rightarrow \boxed{\sigma \approx 1.31659}$$

(d) Use the standard deviation to determine the unusually large values of x.

$$\mu + 2\sigma \approx 1.814 + 2(1.317) = 4.448$$

ANY X VALUE GREATER THAN 4 IS UNUSUALLY LARGE.

(e) Use the 5% rule to determine the unusually large values of x.

SINCE $P(x \geq 4) = 0.086 > 0.05$,

4 IS NOT UNUSUAL.

However, $P(x \geq 5) = 0.039$, so $\boxed{\text{5 or more are unusual.}}$

THIS AGREES WITH PART (d).

2. (3 points) The values of the random variable x and its probabilities are shown below.

x	0	1	2	3	4	5	6
$P(x)$	0.03	?	0.18	0.35	0.38	?	0.01

Find values for $P(1)$ and $P(5)$ so that the table describes a probability distribution **and** both $x = 1$ and $x = 5$ are unusual.

$0.03 + 0.18 + 0.35 + 0.38 + 0.01 = 0.95$

WE MUST HAVE

$P(1) + P(5) = 0.05$

$P(0) + P(1) < 0.05$

$P(5) + P(6) < 0.05$

$\Longrightarrow$ CHOOSE $P(1) = 0.015$ AND $P(5) = 0.035$

3. (9 points) On average, an American uses 123 gallons of water daily, with a standard deviation of 21 gallons. Thirty-five Americans are randomly selected.

(a) What is the probability that mean water usage of the sample is between 120 gallons and 123 gallons?

SINCE $N = 35 > 30$, SAMPLING IS APPROX NORMAL

$\mu_{\bar{x}} = 123$

$\sigma_{\bar{x}} = \dfrac{21}{\sqrt{35}}$

$P(120 \leq \bar{x} \leq 123)$

$= \text{normalcdf}\left(120, 123, 123, \frac{21}{\sqrt{35}}\right)$

$= \boxed{0.3010}$

(b) What would be an unusually small sample mean?

$\mu_{\bar{x}} - 2\sigma_{\bar{x}} = 123 - 2\left(\frac{21}{\sqrt{35}}\right) \approx \boxed{115.9}$

ANY MEAN BELOW 115.9 IS UNUSUALLY SMALL.

(c) Are the sampling means normally distributed? Explain?

YES, THEY ARE APPROXIMATELY NORMAL BECAUSE $N = 35 > 30$.

4. (10 points) In the state of Illinois, there are about 79 traffic fatalities per month.

   (a) What is the probability that there are exactly 79 traffic fatalities in any given month?

POISSON

$\mu = 79$

$$P(x = 79) = \text{poissonpdf}(79, 79)$$

$$= \boxed{0.0448}$$

   (b) In any given month, what is the probability that there are more than 88 traffic fatalities?

$$P(x > 88) = 1 - P(x \le 88)$$

$$= 1 - \text{poissoncdf}(79, 88)$$

$$= \boxed{0.1430}$$

   (c) In July 2011, there were 101 traffic fatalities. Is this an unusually large number of fatalities? Explain.

$$\mu + 2\sigma = 79 + 2\sqrt{79} = 96.78$$

ANY NUMBER MORE THAN THIS IS UNUSUAL

$\boxed{\text{101 IS UNUSUAL}}$

   (d) In January 2011, there were 67 traffic fatalities. Is this an unusually small number of fatalities?

$$\mu - 2\sigma = 79 - 2\sqrt{79} = 61.22$$

FEWER THAN THIS IS UNUSUAL.

$\boxed{\text{67 IS NOT UNUSUAL}}$

5. (3 points) In the recent election, 66.4% of voters supported increasing the state's minimum hourly wage. In random sample of 23 voters, what is the probability that at least 17 supported the increase?

BINOMIAL

$p = 0.664$

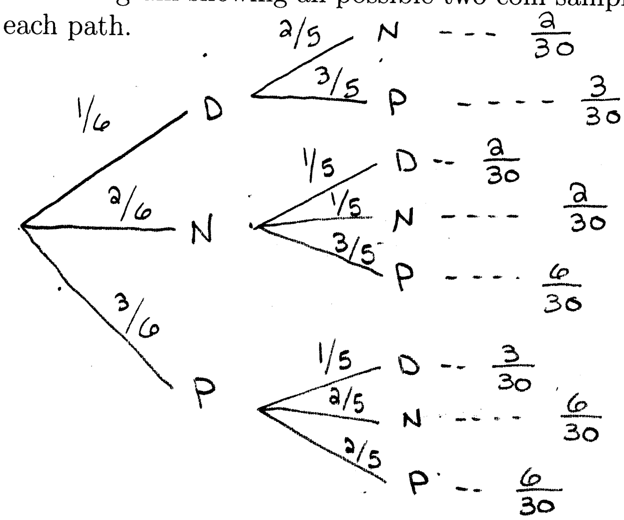
$$P(x \ge 17) = 1 - P(x \le 16)$$

$$= 1 - \text{binomcdf}(23, 0.664, 16)$$

$$= \boxed{0.3006}$$

6. (12 points) A jar contains one dime, two nickels, and three pennies. Two coins are randomly selected without replacement.

(a) Sketch the tree diagram showing all possible two-coin samples. Include the probability of each path.



(b) Find the median value of each two-coin sample. Then summarize the sampling distribution for the medians in a probability distribution table.

X (MEDIAN)	1 (PP)	3 (PN)	5 (NN)	5.5 (DP)	7.5 (DN)
P(x)	$\frac{6}{30}$	$\frac{12}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{4}{30}$

(c) Find the mean of the sample medians.

$$\mu = 1\left(\frac{6}{30}\right) + 3\left(\frac{12}{30}\right) + 5\left(\frac{2}{30}\right) + 5.5\left(\frac{6}{30}\right) + 7.5\left(\frac{4}{30}\right)$$

$$= \boxed{3.8\overline{3}}$$

(d) Find the population median. (There are 6 coins in the population.)

$$1, 1, \underbrace{1, 5}, 5, 10$$

$$\frac{1+5}{2} = \frac{6}{2} = \boxed{3}$$

(e) Do the sample medians target the population median? Explain.

No, $3.8\overline{3} \neq 3$

7. (9 points) In a survey of 1017 U.S. adults, 93% said they would write a college application essay for their child.

   (a) Construct a 99% confidence interval estimate for the true population proportion of adults who would write their child's essay.

93% OF 1017 ≈ 946

C.I. IS (0.9096, 0.95077)

1-PropZInt

X = 946

n = 1017

C-Level = 0.99

   (b) Using the point estimate $\hat{p} = 0.93$, determine the sample size required to have a margin of error of ± 0.03.

$\alpha = 0.01$

$\alpha/2 = 0.005$

$z_{\alpha/2} = \text{invNorm}(0.995)$

$= 2.576$

$$N = \frac{(2.576)^2(0.93)(0.07)}{(0.03)^2} \approx \boxed{480}$$

   (c) After discussing college admission requirements with a large group of parents, a high school advisor found that 85% of parents said **they would never** write an essay for their child. What do you think about this result?

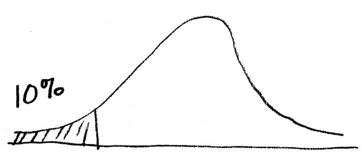
85% WOULD NOT

⇓

15% WOULD

SINCE 15% IS FAR OUTSIDE THE C.I. ESTIMATE, I WOULD SAY THAT A GREAT MANY OF THESE PARENTS ARE LYING.

8. (3 points) The mean lifetime of a wristwatch is 32 months, with a standard deviation of 5 months. If the distribution is normal, for how many months should a guarantee be made if the manufacturer does not want to exchange more than 10% of the watches?



invNorm (0.10, 32, 5)

$\approx \boxed{25.6 \text{ MONTHS}}$

9. (9 points) The numbers of calories in 1.5-ounce chocolate bars are normally distributed with mean 225 and standard deviation 9.

   (a) What is the probability that a randomly selected chocolate bar has exactly 225 calories?

   PROBABILITY OF AN EXACT VALUE IN A CONTINUOUS DISTRIBUTION IS $\boxed{0}$.

   (b) In a sample of 20 chocolate bars, about how many will contain more than 235 calories?

   20 × normalcdf(235, 99999, 225, 9)

   ≈ 2.665

   ABOUT 3 BARS

   (c) What is the probability that a randomly selected chocolate bar has fewer than 210 or more than 240 calories?

   normalcdf(−99999, 210, 225, 9) + normalcdf(240, 99999, 225, 9)

   ≈ 0.0956

10. (9 points) Fifty-five percent of parents believe it is appropriate to lie to their children about Santa Claus. A sample of 25 parents is selected at random. Let x represent the number of parents in the sample who believe lying about Santa is appropriate.

   BINOMIAL

   $N = 25$

   $p = 0.55$

   $q = 0.45$

   (a) What is the mean value of x?

   $$\mu = 25(0.55) = \boxed{13.75}$$

   (b) What is the standard deviation in the values of x?

   $$\sigma = \sqrt{25(0.55)(0.45)} \approx 2.49$$

   (c) What are the cutoff values for the unusually small and unusually large values of x?

   $$\mu - 2\sigma \approx 8.77$$

   $$\mu + 2\sigma \approx 18.73$$