Math 153 - Final Exam

December 9, 2014

Name Key Score

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Suppose A and B are events such that $P(A) = 0.56$, $P(A \cup B) = 0.72$, and $P(A \cap B) = 0.26$.

   (a) Compute $P(B)$.

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   $0.72 = 0.56 + \square - 0.26$

   0.42

   $\boxed{P(B) = 0.42}$

   (b) Compute $P(\overline{A})$.

   $1 - 0.56 = \boxed{0.44}$

   (c) Compute $P(A|B)$. $= \dfrac{P(A \cap B)}{P(B)} \approx \boxed{0.619}$

   (d) Are A and B independent? Explain.

   $\boxed{\text{No,}}$ Because $P(A) \neq P(A|B)$.

   $0.56 \neq 0.619$

   (e) What are the odds against A?

   $\dfrac{P(\bar{A})}{P(A)} = \dfrac{0.44}{0.56} = \dfrac{44}{56} = \boxed{\dfrac{11}{14}}$

2. The frequency distribution shown below gives the daily high temperatures (in °F) last year in Cleveland, OH.

CLASS WIDTH IS 11

High Temp (°F)	Frequency
20–30	19
31–41	43
42–52	68
53–63	69
64–74	74
75–85	68
86–96	24

365 DAYS

(a) (2 points) What are the class midpoints?

$\frac{20+30}{2} = 25$ $\boxed{25, 36, 47, 58, 69, 80, 91}$

(b) (4 points) Use class midpoints to estimate the (weighted) mean temperature.

$$\bar{X} \approx \frac{19(25) + 43(36) + 68(47) + 69(58) + 74(69) + 68(80) + 24(91)}{365}$$

$$= \frac{21951}{365} \approx \boxed{60.14°}$$

(c) (3 points) Use class midpoints to estimate the (weighted) median.

365 DAYS $\Rightarrow$ 365 TEMPS $\Rightarrow$ MEDIAN IS 183RD TEMP

$\Rightarrow$ MEDIAN LIES IN CLASS 53°-63°

$\Rightarrow$ $\boxed{\text{MEDIAN} \approx 58°}$

(d) (3 points) Use probability to determine whether 25° is an usually low temperature for Cleveland? Explain.

$$\frac{19}{365} \approx 0.05205 \approx 5.21\% > 5\%$$

25° IS NOT UNUSUAL.

3. (5 points) For Yellowstone's Old Faithful geyser, the mean time between eruptions is 1.55 hr with a standard deviation of 0.11 hr. For Yellowstone's Lone Star geyser, the mean is 3.00 hr with a standard deviation of 0.16 hr. Compute the coefficient of variation (CV) for each geyser. Which geyser's eruption cycle has more variation?

O.F.

$CV = \frac{0.11}{1.55}$

$\approx 7.1\%$

L.S.

$CV = \frac{0.16}{3.00}$

$\approx 5.3\%$

OLD FAITHFUL'S CYCLE HAS MORE VARIATION.

4. (10 points) The numbers below show the points scored by the Chicago Bulls in their 2012–2013 season playoff games (in the order in which they occurred).

$\cancel{89}$ $\cancel{90}$ $\cancel{79}$ 142 $\cancel{91}$ $\cancel{92}$
$\cancel{99}$ $\cancel{93}$ $\cancel{78}$ $\cancel{94}$ $\cancel{65}$ $\cancel{91}$

Compute the five-number summary, the interquartile range (IQR), and the cutoff values for outliers.

65 78 79 89 90 91 91 92 93 94 99 142

$\frac{79+89}{2} = 84$ $\frac{93+94}{2} = 93.5$

MEDIAN = 91
$Q_1 = 84$
$Q_3 = 93.5$
MIN = 65
MAX = 142

IQR = 93.5 − 84
= 9.5

OUTLIER CUTOFFS:
$84 - 1.5(9.5) = 69.75$
$93.5 + 1.5(9.5) = 107.75$

5. (6 points) Refer to the problem above. Based on the 2012–2013 playoff data, what would be an usually large number of points for the Bulls to score in a playoff game.

$\bar{X} = 91.91\overline{6}$

$S \approx 18.248$

$91.917 + 2(18.248)$
$= 128.413$

MORE THAN 128 WOULD BE UNUSUAL.

6. (12 points) According to a recent Gallup poll[1], only 26% of U.S. urban blacks have confidence in the police. A random sample of 200 urban blacks is selected.

(a) Find the probability that exactly 58 people in the sample have confidence in the police.

BINOMIAL
$p = 0.26$
$n = 200$
$q = 0.74$

$$P(x=58) = \text{binompdf}(200, 0.26, 58)$$
$$= \boxed{0.0393}$$

(b) Find the probability that at least 60 people in the sample have confidence in the police.

$$P(x \geq 60) = 1 - P(x \leq 59)$$
$$= 1 - \text{binomcdf}(200, 0.26, 59)$$
$$= \boxed{0.1143}$$

(c) In that sample, what would be an unusually small number of people who have confidence in the police?

ANY NUMBER LESS THAN 39.59

$$\mu - 2\sigma = np - 2\sqrt{npq}$$
$$= (200)(0.26) - 2\sqrt{200(0.26)(0.74)} \approx 39.59$$

7. (8 points) U.S yearly per capita consumption of fresh apples (in pounds) is approximately normally distributed with mean 9.5 lbs and standard deviation 2.8 lbs.

(a) How many pounds of apples must one eat to be in the 95th percentile of apple-eaters?

NORMAL
$\mu = 9.5$
$\sigma = 2.8$

$$\text{invNorm}(0.95, 9.5, 2.8) \approx \boxed{14.11 \text{ lbs}}$$

(b) In a random sample of 700 Americans, about how many eat less than 9 lbs of apples per year?

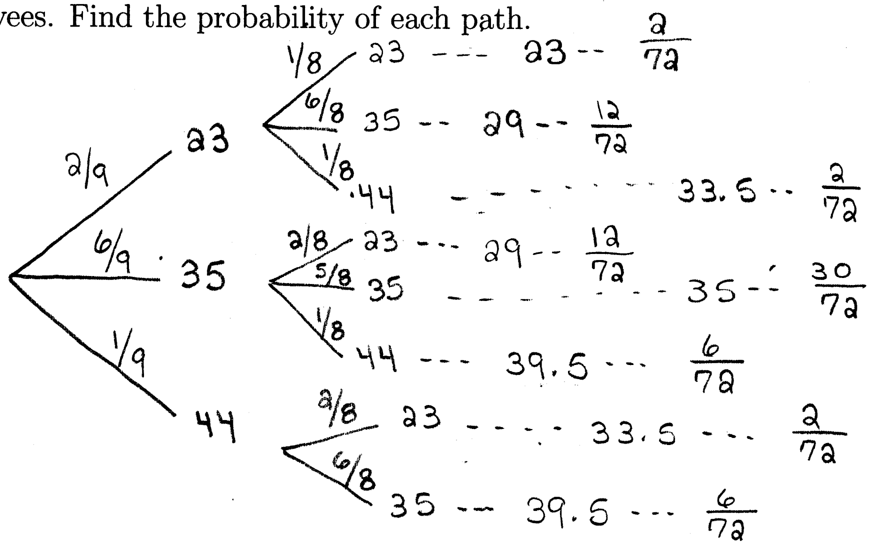
$$700 \times \text{normalcdf}(-99999, 9, 9.5, 2.8)$$
$$\approx 700 \times 0.4291 \approx 300.4$$

ABOUT 300

[1] http://www.gallup.com/poll/179909/urban-blacks-little-confidence-police.aspx

8. A small business has two employees of age 23, six employees of age 35, and one employee of age 44. Two employees are selected at random without replacement. Let the random variable x represent the median age of the two employees.

(a) (8 points) Sketch the probability tree associated with the selection of the two employees. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

23, 29, 33.5, 35, 39.5

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	23	29	33.5	35	39.5
P(x)	$\frac{2}{72}$	$\frac{24}{72}$	$\frac{4}{72}$	$\frac{30}{72}$	$\frac{12}{72}$

(d) (6 points) Find the mean value of x.

$$\mu = \frac{2(23) + 24(29) + 4(33.5) + 30(35) + 12(39.5)}{72} = \frac{2400}{72} = \boxed{33\tfrac{1}{3}}$$

(e) (4 points) Find the median age of the employees. Do the sample medians target the population median?

23, 23, 35, 35, 35, 35, 35, 35, 44

$\boxed{\text{MEDIAN} = 35}$

No, $35 \neq 33\frac{1}{3}$

9. (8 points) Illinois experiences about 37.6 tornadoes per year.

(a) In any given year, what is the probability of there being fewer than 33 tornadoes?

POISSON
$\mu = 37.6$

$$P(x < 33) = P(x \leq 32) = \text{poissoncdf}(37.6, 32)$$

$$\approx \boxed{0.2050}$$

(b) In a given year, what would be an unusually large number of tornadoes?

$$\mu + 2\sigma = \mu + 2\sqrt{\mu} = 37.6 + 2\sqrt{37.6} \approx \boxed{49.86}$$

50 OR MORE ARE UNUSUAL

10. (12 points) From 1962 to 1981, weights of U.S. pennies were approximately normally distributed with mean 3.11 g and standard deviation 0.029 g. A random sample of 12 pennies is obtained.

(a) What are the mean and standard deviation of the sampling distribution?

$$\mu_{\bar{x}} = 3.11$$

$$S_{\bar{x}} = \frac{S}{\sqrt{N}} = \frac{0.029}{\sqrt{12}} \approx 0.00837$$

(b) What is the probability that the sample mean is greater than 3.13 g?

$$\text{normalcdf}(3.13, 99999, 3.11, 0.029/\sqrt{12})$$

$$\approx \boxed{0.0084}$$

(c) What would be an usually small sample mean?

$$3.11 - 2(0.00837) \approx \boxed{3.093}$$

11. (12 points) A recent Gallup poll of 1500 U.S. adults shows that President Obama's approval rating is 43%.

(a) Construct a 99% confidence interval estimate for the true proportion of U.S. adults who approve of the president.

1-PropZInt

$X = 0.43 \cdot 1500 = 645$

$n = 1500$, C-Level: 0.99

$(0.39707, 0.46293)$

WE CAN BE 99% CONFIDENT THAT THE TRUE pop. PROPORTION IS BETWEEN 39.7% AND 46.3%

(b) Use the proportion estimate found by the Gallup poll to determine the sample size required to have a 3% margin of error at the 99% confidence level.

$$N = \frac{[z_{\alpha/2}]^2(0.43)(0.57)}{[0.03]^2} = \frac{(2.576)^2(0.43)(0.57)}{0.0009} = 1807.14\ldots$$

$\boxed{\text{CHOOSE } N = 1808}$

12. (15 points) Scores on a statistics final exam are approximately normally distributed. A random sample of ten scores is shown below.

$$98.5, 131, 127, 141.5, 150, 66.5, 98.5, 84, 104, 108.5$$

The statistics professors claim that the mean score is greater than 95. Test the claim made by the professors at the level $\alpha = 0.05$.

(a) State the null and alternative hypotheses.

CLAIM: $\mu > 95$

Opp. CLAIM: $\mu \leq 95$

$\boxed{H_0: \mu = 95 \quad H_1: \mu > 95}$

(b) Compute the test statistic.

T-Test WITH DATA

$\mu_0 : 95$

$\mu : > \mu_0$

$\boxed{t \approx 1.924}$

(c) Find the P-value and draw a conclusion about professors' claim.

$\boxed{\text{P-VALUE} \approx 0.043}$

P-VALUE $< \alpha = 0.05 \Rightarrow$ WE REJECT THE NULL HYPOTHESIS IN FAVOR OF THE PROFESSORS' CLAIM.

13. (16 points) The table below shows the values of a share of common stock of Dr. Pepper Snapple Group, Inc. (DPS) at selected days since the beginning of 2014.

x (Days)	0	60	120	150	180	240	270	330
y (Value in \$)	48.72	52.11	55.42	57.70	58.58	63.03	64.31	73.03

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between x and y.

$\boxed{r \approx 0.9808}$

THIS INDICATES A STRONG LINEAR RELATIONSHIP.

(b) Compute the corresponding P-value and draw a conclusion about the existence of a linear relationship

$H_0: \rho = 0$

$H_1: \rho \neq 0$

$\boxed{\text{P-VALUE} = 0.0000176}$

VERY SMALL P-VALUE INDICATES STRONG LINEAR RELATIONSHIP.

(c) Find the regression equation.

$\hat{y} = 0.06824x + 47.59720$

- OR -

$\hat{y} \approx 0.07x + 47.60$

(d) Use your regression equation to predict the value of the stock on day 200.

WHEN $x = 200$,

$\hat{y} \approx 61.24$