Math 153 - Test 2
October 15, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) The numbers of tornadoes in Illinois for each year from 2003 to 2014 are shown below.

$$120, 80, 21, 123, 23, 49, 52, 48, 73, 31, 54, 48$$

(a) Find the range and the sample standard deviation.

RANGE = 123 - 21 = 102

FROM CALCULATOR ...: $s \approx 33.545$

(b) Find the median, quartiles, and the interquartile range.

FROM CALCULATOR ... $Q_1 = 39.5$, MED = 50.5, $Q_3 = 76.5$

$IQR = 76.5 - 39.5 = 37$

(c) Compute the cut-off values for outliers.

$Q_1 - 1.5(IQR) = 39.5 - 1.5(37) = -16$

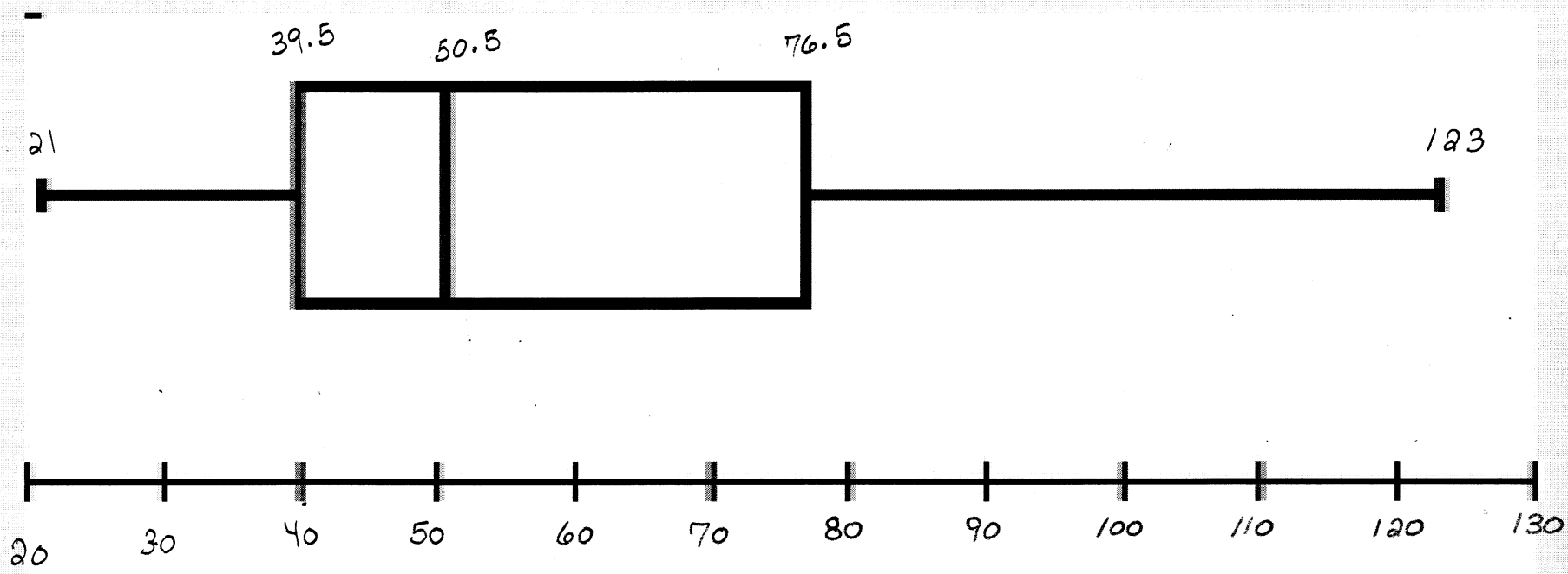
$Q_3 + 1.5(IQR) = 76.5 + 1.5(37) = 132$

THERE ARE NO OUTLIERS.

(d) Sketch the modified boxplot.

SEE ATTACHED SHEET.

NORMAL FLOAT AUTO REAL RADIAN MP



2. (12 points) In a large sample of men, the mean height was 69.8 in with a standard deviation of 2.9 in. The mean weight was 176 lbs with a standard deviation of 30 lbs.

(a) Compute z-scores to determine which is "relatively" greater, 75.6 in or 232 lbs.

HEIGHT...

$$z = \frac{75.6 - 69.8}{2.9} = \boxed{2}$$

WEIGHT...

$$z = \frac{232 - 176}{30} \approx \boxed{1.87}$$

THE HEIGHT IS RELATIVELY GREATER

(b) Compute the coefficient of variation for the heights and the weights. Is there more spread in the heights or weights?

HEIGHT...

$$\frac{2.9}{69.8} \approx 0.0415 = \boxed{4.15\%}$$

WEIGHT...

$$\frac{30}{176} \approx 0.1705 = \boxed{17.05\%}$$

MUCH GREATER VARIATION IN WEIGHTS

(c) Based on the sample data, at what weight should a man be considered unusually light?

$$\bar{X} - 2s = 176 - 2(30) = \boxed{116 \text{ lbs}}$$

3. (6 points) A fair, six-sided die is rolled one time. Let A be the event of rolling an even number, and let B be the event of rolling a multiple of 3. Compute $P(A|B)$ and $P(B|A)$. Be sure to indicate which is which.

$P(A|B)$ = PROB OF ROLLING EVEN GIVEN $\{3,6\}$

$$= \boxed{\frac{1}{2}}$$

$P(B|A)$ = PROB OF ROLLING MULTIPLE OF 3 GIVEN $\{2,4,6\}$

$$= \boxed{\frac{1}{3}}$$

4. (10 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

(a) Compute $P(A)$.

$1 - 0.52 = \boxed{0.48}$

(b) Compute $P(A \cap B)$.

$= P(A) + P(B) - P(A \cup B) = 0.48 + 0.55 - 0.766$

$= \boxed{0.264}$

(c) Compute $P(B|A)$.

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.264}{0.48} = \boxed{0.55}$

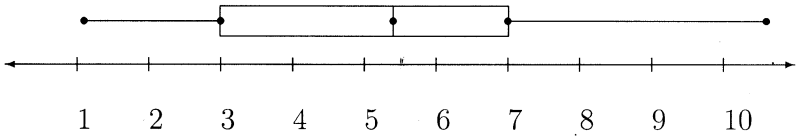
(d) Are A and B independent? Explain.

$\boxed{\text{YES,}}$ BECAUSE $P(B) = P(B|A)$

(e) What are the odds in favor of A?

$\frac{0.48}{0.52} = \frac{48}{52} = \boxed{\frac{12}{13}}$

5. (6 points) The boxplot shown below describes a certain collection of data. Find approximate values for the median, first and third quartiles, and the interquartile range. Based on your approximations, what would be the cutoff values for outliers?



$Q_1 = 3$

$MED = 5.4$

$Q_3 = 7$

$IQR = 7 - 3 = 4$

$Q_1 - 1.5(IQR)$

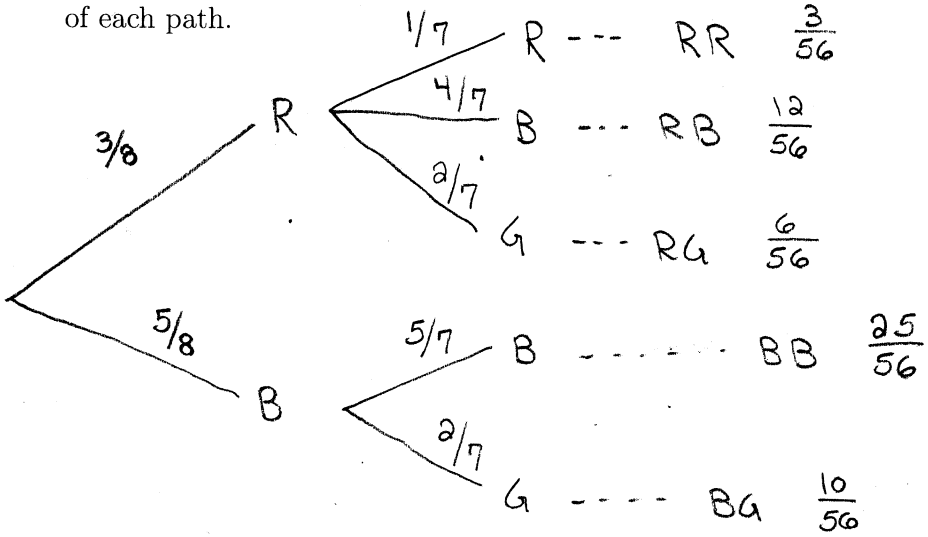
$= 3 - 1.5(4) = -3$

$Q_3 + 1.5(IQR)$

$= 7 + 1.5(4) = 13$

6. (10 points) Box One contains 3 red marbles and 5 blue marbles. Box Two contains 4 blue marbles and 2 green marbles. A marble is selected at random from Box One and placed into Box Two. Then a marble is selected at random from Box Two.

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) What is the probability of drawing a blue marble from Box Two?

RB + BB

$$\frac{12}{56} + \frac{25}{56} = \boxed{\frac{37}{56}}$$

(c) What is the probability of drawing a blue marble from Box One or a green marble from Box Two?

BB + BG + RG

$$\frac{25}{56} + \frac{10}{56} + \frac{6}{56} = \boxed{\frac{41}{56}}$$

(d) What is the probability of drawing a blue marble from Box Two given that a blue marble was drawn from Box One?

$$\boxed{\frac{5}{7}}$$

7. (12 points) The numbers of students at a certain college are described in the table below.

	Female	Male	
Part-time	2112	1408	3520
Full-time	1746	1164	2910
	3858	2572	

2112 + 1408 + 1746 + 1164 = 6430

A college student is selected at random.

(a) What is the probability that the student is a female?

$$\frac{2112+1746}{6430} = \boxed{\frac{3858}{6430} = 60\%}$$

(b) What is the probability that the student is a part-time student?

$$\frac{2112+1408}{6430} = \boxed{\frac{3520}{6430} \approx 54.7\%}$$

(c) What is the probability that the student is a male student or a full-time student?

$$\frac{1408+1164+1746}{6430} = \boxed{\frac{4318}{6430} \approx 67.2\%}$$

(d) What is the probability that the student is a female, part-time student?

$$\boxed{\frac{2112}{6430} \approx 32.8\%}$$

(e) What is the probability that the student is a female given that the student is part-time?

$$\frac{2112}{2112+1408} = \boxed{\frac{2112}{3520} = 60\%}$$

(f) Are being a female student and being a part-time student independent events? Use some of your results from above to support your answer.

YES, BECAUSE PROB IN PART (a) = PROB IN PART (e)

8. (4 points) When John went to work at the liquor store, he wondered how good he was at guessing customers' ages. For the first 38 customers, John guessed correctly 20 times.

   (a) Estimate the probability that John guesses an age correctly.

$$\frac{\#\text{ OF CORRECT GUESSES}}{\text{TOTAL \#}} = \boxed{\frac{20}{38}}$$

   (b) Is your estimate a theoretical, experimental, geometric, or subjective probability?

9. (6 points) A fair, six-sided die is rolled three times. What is the probability that at least one 5 is rolled?

$$1 - \text{PROB OF NO 5's}$$

$$= 1 - \left(\frac{5}{6}\right)^3 = 1 - \frac{125}{216} = \boxed{\frac{91}{216} \approx 42\%}$$

10. (4 points) A nationally administered test has a mean of 500 and a standard deviation of 100. If your standardized score (z-score) on the test was 1.8, what was your actual test score?

$$1.8 = \frac{X - 500}{100} \Rightarrow 180 = X - 500$$

$$\boxed{X = 680}$$

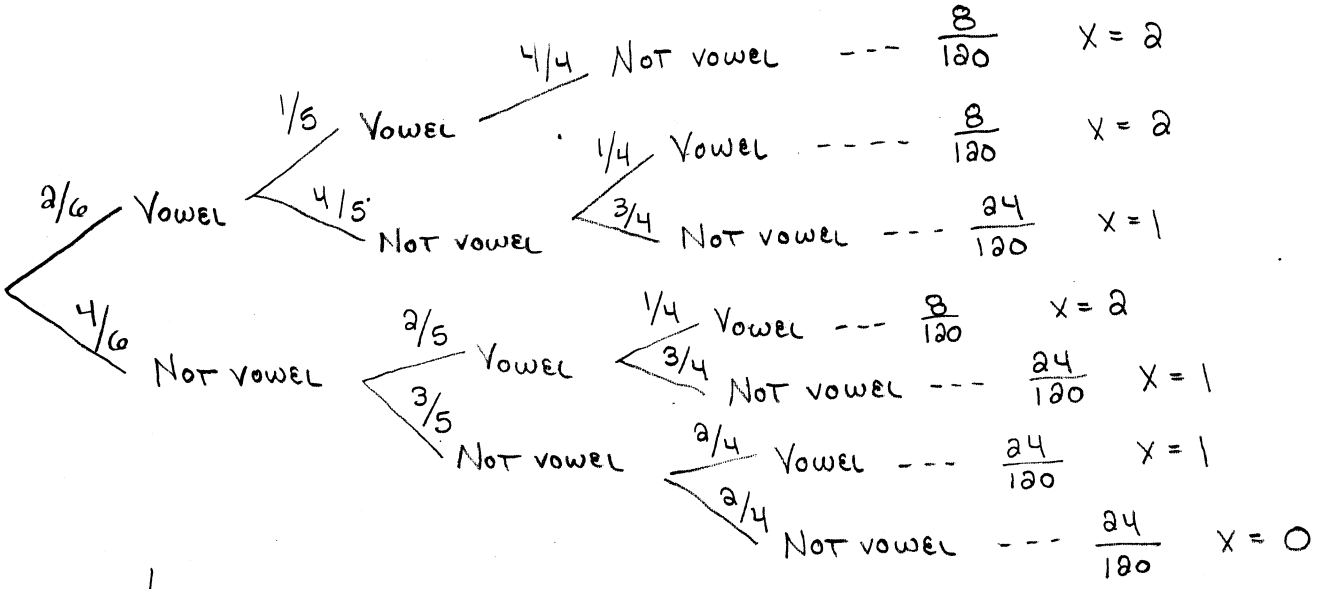
11. (3 points) If the odds against Wicked Strong winning the race are 5 to 4, what is the probability of her winning?

$$\text{ODDS IN FAVOR ARE } \frac{4}{5}$$

$$\Rightarrow \text{PROB IS } \boxed{\frac{4}{9}}$$

12. (Take home portion—15 points) Three letters are selected at random without replacement from the word $ASSESS$. Let the random variable x represent the total number of vowels selected. Sketch the probability tree diagram for the experiment. (Include the probabilities of the paths.) Then determine the probability distribution for x and compute the mean and standard deviation for the variable x.

$X = 0, 1, 2$



X	$P(x)$
0	$24/120$
1	$72/120$
2	$24/120$

$$\mu = 0\left(\frac{24}{120}\right) + 1\left(\frac{72}{120}\right) + 2\left(\frac{24}{120}\right)$$

$$= \frac{120}{120} = \boxed{1}$$

$$\sigma^2 = 0\left(\frac{24}{120}\right) + 1\left(\frac{72}{120}\right) + 4\left(\frac{24}{120}\right) - 1$$

$$= 0.4$$

$$\sigma = \sqrt{0.4} \approx \boxed{0.632}$$