Math 153 - Test 3a

November 12, 2015

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Seventy percent of U.S. veterans voted in the 2012 presidential election. If 50 veterans are selected at random, what is the probability that no fewer than 40 voted in the election?

BINOMIAL...

$p = 0.70$

$n = 50$

$$P(x \geq 40) = 1 - P(x \leq 39)$$

$$= 1 - \text{binomcdf}(50, 0.7, 39) \approx \boxed{0.0789}$$

2. (9 points) On average, there are 13.75 avalanche fatalities in the U.S. each year.

   (a) In any given year, what is the probability that there are no more than 10 avalanche fatalities?

POISSON...

$\mu = 13.75$

$$P(x \leq 10) = \text{poissoncdf}(13.75, 10) \approx \boxed{0.1928}$$

   (b) In any given year, what is the probability that there are exactly 12 fatalities?

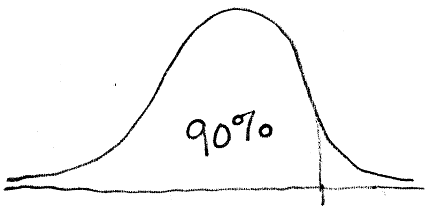
$$P(x = 12) = \text{poissonpdf}(13.75, 12) \approx \boxed{0.1018}$$

   (c) In 1999 and again in 2008, there were 26 avalanche fatalities. Is this an usually large number of fatalities? Explain.

$$\mu + 2\sigma = \mu + 2\sqrt{\mu} \approx 21.17$$

YES, 26 IS UNUSUALLY LARGE.

3. (4 points) Patient recovery times after a certain medical procedure are normally distributed with mean 5.3 days and standard deviation 2.1 days. If a recovery time exceeds the 90th percentile, the patient is given an extensive follow-up examination. What recovery time is at the 90th percentile?



$X = \text{invNorm}(0.90, 5.3, 2.1) \approx \boxed{8.0 \text{ DAYS}}$

4. (6 points) In a recent Gallup Poll survey of 1015 U.S. adults, 223 said they had credit card information stolen within the last year. Find a 95% confidence interval estimate for the population proportion of adults who had credit card information stolen. Give an interpretation of your result in a complete sentence.

$n = 1015$

$X = 223$

C-LEVEL: 0.95

95% C.I. ESTIMATE IS

$(0.194, 0.245)$

WE CAN BE 95% CONFIDENT THAT THE TRUE POPULATION PROPORTION OF ADULTS WHO HAVE HAD CREDIT CARD INFO STOLEN IS BETWEEN 19.4% AND 24.5%

5. (6 points) According to the U.S. Census Bureau, in 2014, 28.3% of single-race American Indians and Alaska Natives were in poverty. This is the highest poverty rate of any race group in America. If 200 people in this race group are randomly selected, about how many will be in poverty? What is the standard deviation in samples of this size?

BINOMIAL...

$p = 0.283$

$n = 200$

$\mu = np = 56.6$

$\sigma = \sqrt{npq} \approx 6.4$

ABOUT 56.6 WILL BE IN POVERTY, WITH A STD. DEV OF 6.4

6. (10 points) In the early 19th century, Scottish militiamen had chest sizes that were normally distributed with mean 39.83 in and standard deviation 2.05 in.

(a) The records associated with 5738 militiamen have been found. About how many men in that group had chest sizes between 39 in and 41 in?

NORMAL...

$\mu = 39.83$

$\sigma = 2.05$

$P(39 < x < 41) =$

normalcdf $(39, 41, 39.83, 2.05) \approx 0.3731$

37.31% of 5738 $\approx \boxed{2141}$

(b) What is the probability that a randomly selected militiaman had a smaller chest than 35 in?

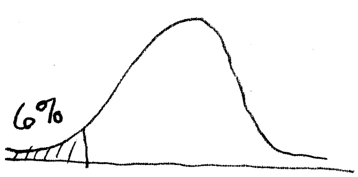
$P(x < 35) =$ normalcdf $(-99999, 35, 39.83, 2.05)$

$\approx \boxed{0.0092}$

(c) What is the probability that a randomly selected militiaman had a chest that measured exactly 42 in?

$P(x = 42) = \boxed{0}$

7. (4 points) The mean lifetime of a certain kind of wristwatch is 36 months, with a standard deviation of 6 months. If the distribution is normal, for how many months should a guarantee be made if the manufacturer does not want to exchange more than 6% of the watches?



$X =$ invNorm $(0.06, 36, 6) \approx \boxed{26.7 \text{ MONTHS}}$

8. (12 points) U.S. adult women have weights that are approximately normally distributed with mean 165.0 lb and standard deviation 45.6 lb. At a Women in Government convention, woman participants are randomly placed into groups of size 14.

(a) One of the groups takes an elevator to the top floor of the convention center. What is the probability that the mean weight of the group exceeds 179 lb (thus overloading the elevator)?

NORMAL/SAMPLING MEAN ...

$\mu_{\bar{x}} = 165$

$\sigma_{\bar{x}} = \dfrac{45.6}{\sqrt{14}}$

$P(\bar{x} > 179)$

$= \text{normalcdf}\left(179, 99999, 165, \dfrac{45.6}{\sqrt{14}}\right)$

$\approx \boxed{0.1253}$

(b) What would be an unusually small mean weight for a group?

$\mu_{\bar{x}} - 2\sigma_{\bar{x}} = 165 - 2\left(\dfrac{45.6}{\sqrt{14}}\right)$

$\approx \boxed{140.6 \text{ lb}}$

(c) Explain how you know that the sample means in this problem are normally distributed.

THE BACKGROUND DISTRIBUTION OF WEIGHTS IS NORMALLY DISTRIBUTED. BY CLT #2, THE SAMPLING MEANS ARE NORMAL.

9. (6 points) According to a Runner's World magazine poll of 432 people, 21% of people cut their toast diagonally. Find a 90% confidence interval estimate for the true population proportion of people who cut their toast diagonally. Give an interpretation of your result in a complete sentence.

$n = 432$

$X = 21\%$ OF $432 \approx 91$

C-LEVEL: 0.90

90% C.I. ESTIMATE IS

(0.178, 0.243)

WE CAN BE 90% CONFIDENT THAT THE TRUE POP. PROPORTION OF DIAGONAL TOAST CUTTERS IS BETWEEN 17.8% AND 24.3%

10. (4 points) On average, an American uses 123 gallons of water daily, with a standard deviation of 21 gallons. Random samples of 10 Americans are selected and the sample means are computed. Are the sample means necessarily normally distributed? Explain.

NO, THE BACKGROUND DISTRIBUTION IS NOT NECESSARILY NORMAL AND THE SAMPLE SIZE IS SMALL ($10 < 30$). THE CLT DOES NOT APPLY.

11. (4 points) Tom wants to compute a 98% confidence interval. What should be his value of $z_{\alpha/2}$?

$\alpha = 0.02$

$\frac{\alpha}{2} = 0.01$

$z_{\alpha/2} = \text{invNorm}(0.99) \approx \boxed{2.326}$

12. (5 points) According to a survey conducted by the New York Times Magazine, 42% of Americans would go back in time and kill Adolf Hitler as a baby if they could. In a sample of 20 Americans, what is the probability that 10 would be willing to kill baby Hitler?

BINOMIAL...

$p = 0.42$

$n = 20$

$P(x = 10) =$

$\text{binompdf}(20, 0.42, 10) \approx \boxed{0.1359}$

13. (5 points) Women have head circumferences that are normally distributed with mean 22.65 in and standard deviation 0.80 in. What percentage of women will fit hats smaller than 21 in or larger than 24 in?

NORMAL...

$\mu = 22.65$

$\sigma = 0.80$

$P(x < 21) + P(x > 24)$

$= 1 - P(21 < x < 24)$

$= 1 - \text{normalcdf}(21, 24, 22.65, 0.80)$

$\approx \boxed{0.0653}$

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Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM. NO CREDIT WILL BE GIVEN FOR GROUP WORK.

1. (10 points) The *harmonic mean* of a set of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers. In symbols, the harmonic mean of $x_1, x_2, \ldots, x_n$ is given by

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}.$$

(a) Find the harmonic mean of the numbers 1, 2, 3.

$$H = \frac{3}{1 + \frac{1}{2} + \frac{1}{3}} = 1.6\overline{3}$$

(b) Two numbers are selected at random with replacement from set $\{1, 2, 3\}$. List all possible two-number samples (there are nine) along with the harmonic mean of each sample.

(1,1) --- 1
(1,2) --- $1.\overline{3}$
(1,3) --- 1.5
(2,1) --- $1.\overline{3}$
(2,2) --- 2
(2,3) --- 2.4
(3,1) --- 1.5
(3,2) --- 2.4
(3,3) --- 3

(c) Determine the sampling distribution for the harmonic means of the samples. Give your answer in the form of a table.

X	1	$1.\overline{3}$	1.5	2	2.4	3
P(x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(d) Find the mean of the sampling distribution.

$$\mu = 1\left(\frac{1}{9}\right) + 1.\overline{3}\left(\frac{2}{9}\right) + \cdots + 3\left(\frac{1}{9}\right) = \frac{247}{135} \approx 1.83$$

(e) Do the sample harmonic means target the population harmonic mean? Explain.

No, $\mu \neq H$. IT WOULD NOT BE WISE TO ESTIMATE THE POP. HARMONIC MEAN BY SAMPLING.

2. (10 points) We would like to determine a 90% confidence interval estimate for the proportion of graduating high school students who enroll at a community college.

(a) Research done at Prairie State College suggests that 17% of local high school graduates enroll at a community college. Given this information, what sample size must be used to obtain a confidence interval with a 4% margin of error?

$$n = \frac{z_{\alpha/2}^2 \hat{p}\hat{q}}{E^2} \qquad n \approx \frac{(1.645)^2(0.17)(0.83)}{0.04^2}$$

$$\approx \boxed{239}$$

$$z_{\alpha/2} = \text{invNorm}(0.95) \approx 1.645$$

(b) If the Prairie State College research described in part (a) is ignored, what sample size would be required?

$$n = \frac{z_{\alpha/2}^2 (0.25)}{E^2} \approx \frac{1.645^2(0.25)}{0.04^2} \approx \boxed{423}$$

(c) In a recent nation-wide survey of 2135 recent high school graduates, it was found that 333 enrolled at a community college. Find a 90% confidence interval for the true population proportion.

$X = 333$

$n = 2135$

C-LEVEL: 0.90

90% C.I. ESTIMATE IS $(0.143, 0.169)$

(d) What is the margin of error in your interval estimate?

$\hat{p} \approx 0.156$ AND $\hat{p} + E \approx 0.169$

$$\Rightarrow E \approx 0.013 = \boxed{1.3\%}$$