Math 153 - Final Exam

December 8, 2015

Name key
Score ______

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) During the last flu season, a number of adults participated in a double-blind study of the effectiveness of a new flu vaccine. The following data were collected.

	Caught the flu	Did not catch the flu
Took vaccine	34	251
Took placebo	100	240

$34+251+100+240=625$

A person from this study is selected at random.

(a) What is the probability that the person took the vaccine?

$$\frac{34+251}{625}=\frac{285}{625}\approx 45.6\%$$

(b) What is the probability that the person caught the flu?

$$\frac{34+100}{625}=\frac{134}{625}=21.44\%$$

(c) What is the probability that the person took the vaccine and caught the flu?

$$\frac{34}{625}=5.44\%$$

(d) What is the probability that the person caught the flu given that he/she took the placebo?

$$\frac{100}{100+240}=\frac{100}{340}\approx 29.41\%$$

(e) Are catching the flu and taking the placebo independent events? Show your work!

No, because prob in (d) $\neq$ prob in (b)

(f) What are the odds against the event that the person took the vaccine?

$$\frac{P(\bar{A})}{P(A)}=\frac{340/625}{285/625}=\frac{340}{285}=\frac{68}{57}$$

2. Historical records regarding the chest sizes of 19th century Scottish militiamen are summarized in the table below.

Chest size (in)	Frequency
33–36	287
37–40	3321
41–44	2054
45–48	76

5738

(a) (2 points) What is the class width?

$37 - 33 = \boxed{4}$

(b) (2 points) What are the class boundaries?

32.5, 36.5, 40.5, 44.5, 48.5

(c) (2 points) What are the class midpoints?

$\frac{33+36}{2} = 34.5$

34.5, 38.5, 42.5, 46.5

(d) (4 points) Use class midpoints to estimate the (weighted) mean chest size.

$$\bar{X} \approx \frac{34.5(287) + 38.5(3321) + 42.5(2054) + 46.5(76)}{5738}$$

$$= \frac{228589}{5738} \approx \boxed{39.8 \text{ in}}$$

(e) (2 points) Use class midpoints to estimate the (weighted) median.

$$\text{MEDIAN} = \frac{2869^{\text{TH}} \text{ VALUE} + 2870^{\text{TH}} \text{ VALUE}}{2} = \boxed{38.5 \text{ in}}$$

(f) (2 points) If a relative frequency histogram was constructed from the data in the table, what would be the height of the bar (or bin) associated with the last class?

$\frac{76}{5738} \approx 0.013$ OR ABOUT 1.3%

3. (6 points) A letter is selected at random from the word *EYJAFJALLAJOKULL*.

   (a) What is the probability of selecting the letter L?

   $\boxed{\frac{4}{16}} = \frac{1}{4}$

   (b) What is the probability of selecting any letter other than an L?

   $1 - \frac{4}{16} = \boxed{\frac{12}{16}} = \frac{3}{4}$

   (c) What are the odds in favor of selecting a vowel (A,E,I,O,U)?

   $\boxed{\frac{6}{10}}$

4. (10 points) Shown below are the numbers of U.S tornado fatalities in the years 1978–1991 (in the order in which they occurred).

53	83	28	24	64	34	122
93	15	59	32	50	53	39

Compute the five-number summary, the interquartile range (IQR), and the cutoff values for outliers.

FROM CALCULATOR...

MIN = 15

Q_1 = 32

MED = 51.5

Q_3 = 64

MAX = 122

IQR = 64 − 32 = 32

CUTOFFS:

32 − 1.5(32) = −16

64 + 1.5(32) = 112

5. (8 points) A recent Gallup poll indicates that 44.9% of U.S. employees are engaged in their work. A random sample of 40 employees is obtained.

   (a) In the sample of size 40, what would be an unusually large number of engaged employees?

   BINOMIAL
   $n = 40$
   $p = 0.449$
   $q = 0.551$

   $\mu = 40(0.449) = 17.96$

   $\sigma = \sqrt{40(0.449)(0.551)} \approx 3.15$

   $\mu + 2\sigma = 24.26$

   25 OR MORE

   (b) What is the probability that at least 25 employees in the sample are engaged in their work?

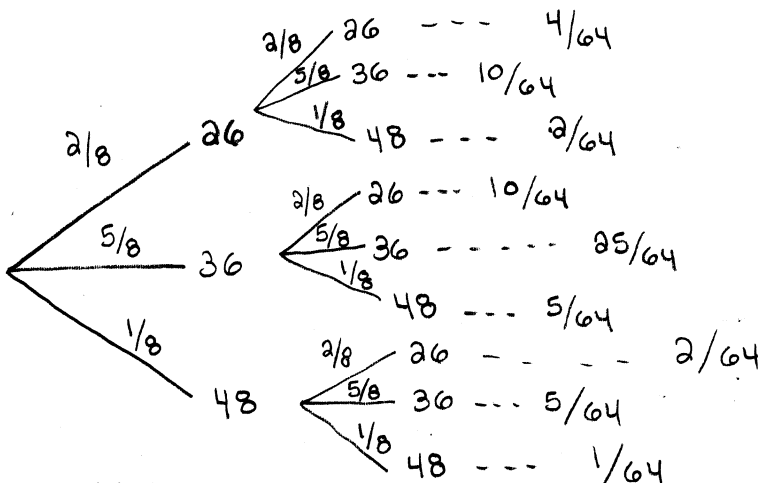
   $P(x \geq 25) = 1 - P(x \leq 24)$

   $= 1 - \text{binomcdf}(40, 0.449, 24)$

   ≈ 0.01898

6. A small business has two employees of age 26, five employees of age 36, and one employee of age 48. Two employees are selected at random **with replacement**. Let the random variable x represent the mean age of the two employees.

(a) (8 points) Sketch the probability tree associated with the selection of the two employees. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

$$\underline{26},\quad \frac{26+36}{2} = \underline{31},\quad \frac{26+48}{2} = \underline{37},\quad \underline{36},\quad \frac{36+48}{2} = \underline{42},\quad \underline{48}$$

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	26	31	36	37	42	48
P(x)	$\frac{4}{64}$	$\frac{20}{64}$	$\frac{25}{64}$	$\frac{4}{64}$	$\frac{10}{64}$	$\frac{1}{64}$

(d) (5 points) Find the mean value of x.

$$\mu_{\bar{x}} = \frac{4}{64}(26) + \frac{20}{64}(31) + \cdots + \frac{1}{64}(48) = \boxed{35}$$

(e) (4 points) Find the mean age of the employees. Do the sample means target the population mean? Explain.

$$\frac{2(26) + 5(36) + 1(48)}{8} = \boxed{35}$$

7. (12 points) Assume that weights at birth are normally distributed with mean 3389 g and standard deviation 540 g.

(a) What is the probability that a randomly selected baby weighed more than 4000 g at birth?

$P(x > 4000) = \text{normalcdf}(4000, 99999, 3389, 540)$

$\approx \boxed{0.1289}$

(b) In a sample of 500 people, about how many had birth weights between 3000 g and 3750 g?

$500 \times \text{normalcdf}(3000, 3750, 3389, 540)$

$\approx \boxed{256}$

(c) What is an usually small birth weight?

$3389 - 2(540) = 2309$

ANY WEIGHT LESS THAN 2309g

8. (8 points) Consider the following probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0.012	0.026	0.020	0.001	0.472	0.002	0.324	0.143

(a) Compute $P(x \geq 4)$.

$0.472 + 0.002 + 0.324 + 0.143 = \boxed{0.941}$

(b) Determine all unusually small values of x.

0 & 1 ARE UNUSUALLY SMALL BECAUSE $P(x \leq 1) = 0.038 < 0.05$

2 IS NOT UNUSUALLY SMALL BECAUSE $P(x \leq 2) > 0.05$

(c) Determine any unusually large values of x.

NONE, BECAUSE $P(x \geq k) > 0.05$ FOR ALL POSSIBLE VALUES OF k.

9. (4 points) Men's heights are normally distributed with mean 69.5 in and standard deviation 2.4 in. What aircraft ceiling height will allow 96% of all men to stand without bumping heir heads?

$$\text{invNorm}(0.96, 69.5, 2.4) \approx \boxed{73.7 \text{ in}}$$

10. (12 points) Contrary to popular reports, the state of Colorado has, on average, 115 clear days per year.

Poisson $\mu = 115$

(a) In any given year, what is the probability of there being exactly 112 clear days?

$$P(x = 112) = \text{Poissonpdf}(115, 112) \approx \boxed{0.0362}$$

(b) In any given year, what is the probability of there being more than 120 clear days?

$$P(x > 120) = 1 - P(x \leq 120)$$
$$= 1 - \text{Poissoncdf}(115, 120) \approx \boxed{0.3000}$$

(c) In a given year, what would be an unusually small number of clear days?

$$115 - 2\sqrt{115} \approx 93.55$$

$\boxed{93 \text{ OR FEWER}}$

11. (6 points) In a large study of the Atkins weight loss program, it was found that the standard deviation in the amount of weight lost over a year was 4.8 lb. What sample size is required to compute a 99% confidence interval estimate for the mean weight loss with a margin of error of 0.25 lb?

$$n = \frac{(z_{\alpha/2})^2 (4.8)^2}{(0.25)^2} = \frac{[\text{invNorm}(0.995)]^2 (4.8)^2}{(0.25)^2}$$

$$\approx 2445.89$$

CHOOSE $\boxed{n = 2446}$

12. (8 points) A random sample of 11 brands of strawberry yogurt gave the following calorie counts per serving:

$$130, 160, 150, 120, 120, 110, 170, 160, 110, 130, 90.$$

Assume the calorie counts come from a normally distributed population. Find a 95% confidence interval estimate for the mean calorie count of all strawberry yogurts. Give an interpretation of your interval in a complete sentence.

TInterval w/ Data

$\Rightarrow$ THE 95% C.I. ESTIMATE

IS $(114.87, 148.77)$.

WE CAN BE 95% CONFIDENT THAT THE MEAN CALORIE COUNT OF THE STRAWBERRY YOGURT POPULATION IS BETWEEN 114.87 AND 148.77

13. (15 points) According to a recent news article, fewer than one-half of Americans favor stricter stricter gun laws. In an October 2015 Gallup poll of 1500 Americans, 55% indicated that they did indeed favor stricter laws. Test the claim made in the news article at the level $\alpha = 0.05$.

(a) State the null and alternative hypotheses.

CLAIM: $p < 0.50$

COUNTER: $p \geq 0.50$

$\longrightarrow$

$H_0: p = 0.50$

$H_1: p < 0.50$

(b) Compute the test statistic.

$X = 0.55(1500) = 825$

$n = 1500$

LEFT-TAILED 1-Prop Z Test

TEST STAT

$Z = 3.87$

(c) Find the P-value and draw a conclusion **about claim in the news article**.

P-VALUE $\approx 0.99995 > \alpha$

$\Rightarrow$ WE DO NOT REJECT H_0.

THE EVIDENCE DOES NOT SUPPORT THE CLAIM OF THE NEWS ARTICLE.

14. (12 points) The following table summarizes some information contained in a 2001 report issued by the Centers for Disease Control and Prevention. For women married at age 25 or older, the table shows the percent (p) of marriages disrupted after t years.

t (in years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
p (in %)	0	2	5	8	11	15	17	19	20	22	24	26	29	30	31	35

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between p and t.

$r \approx 0.993$

THIS IS EVIDENCE OF A STRONG LINEAR RELATIONSHIP BETWEEN p & t.

(b) Compute the corresponding P-value and draw a conclusion about the existence of a linear relationship

P-VALUE $\approx 1.92 \times 10^{-14}$

THIS CAUSES US TO REJECT THE HYPOTHESIS $r = 0$. THEREFORE, THE EVIDENCE SUPPORTS THE EXISTENCE OF A LINEAR RELATIONSHIP.

(c) Find the regression equation.

$\hat{p} = 2.2529t + 1.4779$

(d) Use your regression equation to predict the percent of marriages disrupted after 4.5 years.

WHEN $t = 4.5$,

$\hat{p} \approx 11.6$