Math 153 - Test 2

October 13, 2016

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all statistical computations, but say when you do so.

54 YEARS OF DATA

1. (6 points) Refer to the Illinois tornado data on the attached sheet.

   (a) In 1990 there were 50 tornadoes in Illinois. At what percentile is that number of tornadoes?

   $$\frac{42}{54} = 0.\overline{7} \approx 78\% \qquad 78^{TH} \text{ PERCENTILE}$$

   (b) What number of tornadoes is at the 50th percentile?

   $$50^{TH} \text{ PERCENTILE IS MEDIAN} = \frac{27^{TH} + 28^{TH}}{2} = \frac{29+30}{2} = 29.5$$

   (c) What number of tornadoes is at the 35th percentile?

   $$0.35(54) = 18.9 \Rightarrow 19^{TH} \text{ VALUE} = 22$$

2. (7 points) Over the 25-year period from 1980 through 2004, the annual snowfall recorded at O'Hare Airport averaged 10.9 inches with a standard deviation of 7.5 inches. Over the same time period, Illinois averaged 41.0 tornadoes per year with a standard deviation of 28.3.

   (a) Compute the CV's and determine whether inches of snowfall or numbers of tornadoes have greater relative spread.

   SNOWFALL:

   $$CV = \frac{7.5}{10.9} \approx 68.8\%$$

   TORNADOES:

   $$CV = \frac{28.3}{41.0} \approx 69.0\%$$

   ALMOST THE SAME SPREAD. TORNADOES JUST A BIT MORE SPREAD.

   (b) In this problem, would it have made sense to simply compare the standard deviations or were CV's required? Explain.

   YOU MUST COMPARE THE CVs --- THE VALUES COME FROM VERY DIFFERENT POPULATIONS.

3. (18 points) Suppose a letter is selected at random from the word *kakorrhaphiophobia.*

(a) What is the sample space?

$\{k, a, o, r, h, p, i, b\}$

(b) Are the outcomes in your sample space equally likely? Explain.

NO, THERE ARE MORE h'S THAN k'S IN THE WORD. SO, h'S ARE MORE LIKELY THAN k'S

(c) Choose two outcomes in your sample space and determine their probabilities.

$P(\{h\}) = \frac{3}{18}$ $\quad$ $P(\{k\}) = \frac{2}{18}$

(d) Are the probabilities given above theoretical, experimental, or subjective?

(e) State an event that has probability greater than 0.6, and give the probability of your event.

LET A BE THE EVENT OF SELECTING ANY LETTER EXCEPT i OR o = $\{i, o\}$

$P(A) = \frac{13}{18} = 0.7\overline{2}$

(f) Kate claims that the odds of selecting a vowel are 8/18. Is Kate correct? Explain.

KATE IS NOT CORRECT. SHE COMPUTED A PROBABILITY. THE ODDS ARE $\frac{8}{10}$ OR $\frac{4}{5}$.

4. (10 points) The numbers shown below are the amounts of yearly snowfall, in inches, measured at O'Hare Airport in the years from 1968 to 1979.

~~10.4~~, ~~3.7~~, ~~9.5~~, ~~10.0~~, ~~7.6~~, ~~0.5~~, ~~7.4~~, ~~3.5~~, ~~10.0~~, ~~7.2~~, ~~21.9~~, ~~34.3~~

Compute the five-number summary and the outlier cutoffs. Then sketch the modified boxplot on the graph paper provided.

0.5 3.5 3.7 7.2 7.4 7.6 9.5 10.0 10.0 10.4 21.9 34.3

$Q_1 = \frac{3.7+7.2}{2} = 5.45$ MEDIAN $= \frac{7.6+9.5}{2} = 8.55$ $Q_3 = 10.2$

5-NUM. SUM.
MIN = 0.5
Q_1 = 5.45
MED = 8.55
Q_3 = 10.2
MAX = 34.3

IQR = 10.2 − 5.45
= 4.75

CUTOFFS:
5.45 − 1.5 (4.75) = −1.675
10.2 + 1.5 (4.75) = 17.325

* 21.9 & 34.3 ARE OUTLIERS

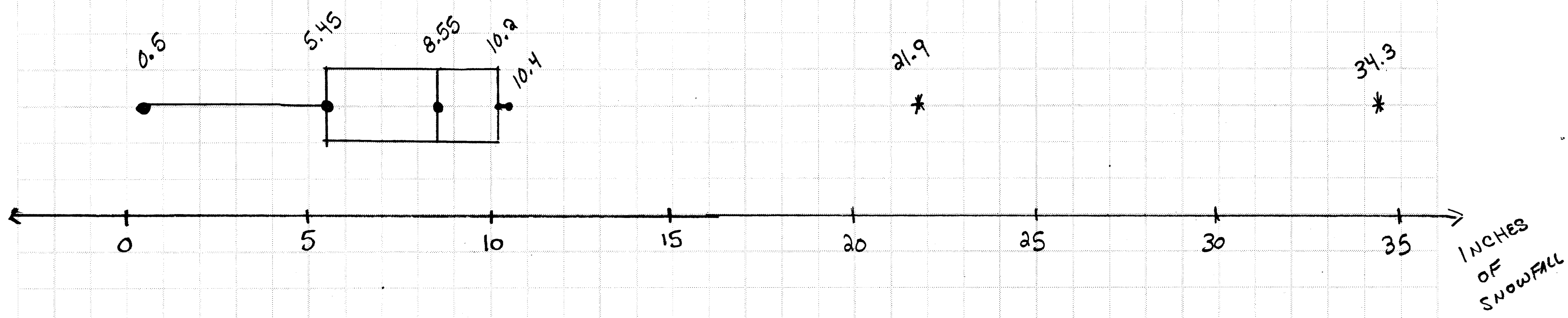
5. (4 points) Lisa took a standardized test and obtained an unusually high score. Nonetheless, quite a few people scored higher than Lisa. Give an example of a z-score that could represent Lisa's score. Explain your reasoning.

$\boxed{2.1}$ THIS IS A LARGE ENOUGH SCORE TO BE UNUSUAL (> 2), BUT NOT SO LARGE AS TO BE RARE. IF THE POPULATION IS APPROXIMATELY NORMAL, ABOUT 2.5% OF SCORES ARE GREATER THAN 2.1

6. (4 points) A PSC student is selected at random. Let Y be the event that the student is taking a math class. Let Z be the event that the student is a male. Are Y and Z disjoint (mutually exclusive)? Explain.

NO, OUR CLASS CONTAINS MANY PSC STUDENTS WHO ARE IN BOTH Y AND Z.

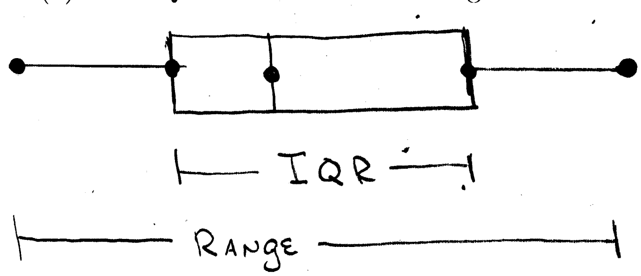
$Y \cap Z \neq \phi$



Boxplot For #4

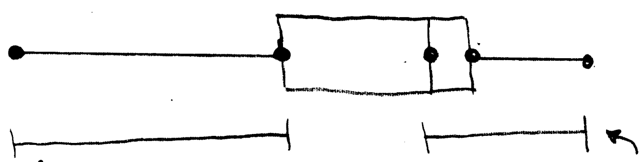
7. (9 points) Think carefully about the about the characteristics of a modified boxplot. For each part of this problem, sketch a boxplot that would correspond to a data set with the given properties.

(a) The IQR is one-half of the range.



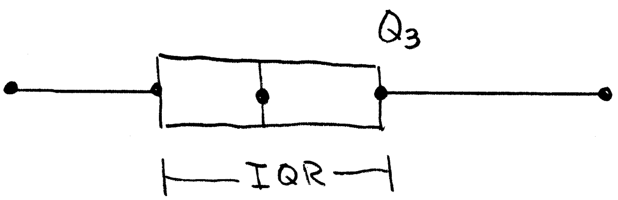
IQR ≈ $\frac{1}{2}$ RANGE

(b) There is more spread in the lower extreme than in the upper half.



THIS LENGTH IS GREATER THAN THIS LENGTH

(c) There is one outlier in the upper extreme.



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MORE THAN $1.5 \times$ IQR ABOVE Q_3

8. (3 points) In the following stem-and-leaf plot, 4 | 5 means 45.

3	1	4				
4	1	2	5			
5	0	0	2	6	7	8
6	3	8	8			
7	0					

Determine the median.

15 VALUES $\Rightarrow$ 8^{TH} VALUE IS MEDIAN

MEDIAN = 52

9. (4 points) A nationally administered test has a mean of 500 and a standard deviation of 100. If your standardized score (z-score) on the test was 1.8, what was your actual test score?

$$1.8 = \frac{X - 500}{100} \Rightarrow 180 = X - 500$$

$$\Rightarrow \boxed{X = 680}$$

10. (8 points) Listed below are the lengths of time (in years) it took for a random sample of college students to earn bachelor's degrees.

4	4	4	4	4	4	4.5	4.5	4.5	4.5
4.5	4.5	6	6	8	9	9	13	13	15

(a) Compute the mean, median, and mode(s)?

MODES ARE 4 & 4.5 ,

$$\bar{X} = \frac{4(6) + 4.5(6) + 6(2) + 8 + 9(2) + 13(2) + 15}{20}$$

$$= 6.5$$

, MEDIAN = $\frac{4.5+4.5}{2}$ = 4.5

(b) Which of the values that you computed above is the best measure of center? Why do you think so?

I THINK THE MODES ARE BEST.

BASED ON THE DATA, THEY CLEARLY REPRESENT HOW MUCH TIME THE "AVERAGE" PERSON SPENDS ON HIS/HER DEGREE.

(c) Compute the sample standard deviation.

$S = 3.5$ (CALCULATOR)

(d) Based on these results, is it unusual for someone to earn a bachelor's degree in 12 years?

$$\bar{X} + 2s = 6.5 + 2(3.5)$$

$$= 13.5$$

$$12 < 13.5$$

$\Rightarrow$ NOT UNUSUAL.

11. (4 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

   (a) Compute $P(A)$.

   $1 - 0.52 = 0.48$

   (b) Compute $P(A \cap B)$.

   $P(A) + P(B) - P(A \cup B)$

   $= 0.48 + 0.55 - 0.766 = 0.264$

12. (2 points) The sample space for a probability experiment is $\{a, b, d, q\}$. Must it be true that $P(\{d, q\}) = 1/2$? Explain.

   NO, THE OUTCOMES IN THE SAMPLE SPACE ARE NOT NECESSARILY EQUALLY LIKELY.

13. (2 points) Mike recently got his driver's license, but he's not so good at driving. When Mike asked to use his mom's car for the third time, she said, "No way! You've run into something both times you've driven it! There is a 100% chance you'll run into something again." What type of probability (theoretical, experimental, subjective, or geometric) did Mike's mother compute?

   2 OUT OF 2 = 100%

   EXPERIMENTAL

14. (6 points) The Insurance Institute for Highway Safety conducted tests with crashes of new cars traveling at 6 mi/h. The costs of damage from a simple random sample are shown below. Compute the sample mean and standard deviation. Based on your result, is damage of \$10,000 unusual? Briefly explain.

\$7448 \$4911 \$9051 \$6374 \$4277

CALCULATOR:

$\bar{X} = 6412.2$

$S \approx 1926.8$

$$\underbrace{\bar{X} + 2s \approx 10265.81}$$

\$10,000 IS NOT UNUSUAL.

IT IS LESS THAN 2 STD DEV'S FROM MEAN.

15. (4 points) In a study of students' homework habits, a professor collected the following data.

	Did homework	Did not do homework
Received A or B	97	32
Received C, D, or F	41	78

A student from this study is selected at random.

(a) What is the probability that the student did homework or received A or B?

$$\frac{97+41+32}{97+41+32+78} = \boxed{\frac{170}{248} \approx 68.5\%}$$

(b) What are the odds against the student doing homework?

$$\text{PROB IS } \frac{97+41}{97+41+32+78} = \frac{138}{248}$$

$$\Rightarrow \text{ODDS AGAINST ARE } \frac{248-138}{138} = \frac{110}{138} = \boxed{\frac{55}{69}}$$

16. (3 points) A collection of weights has mean 178.8 lbs and coefficient of variation 6.5%. Determine the standard deviation.

$$0.065 = \frac{s}{178.8} \Rightarrow \boxed{s = 11.622}$$

17. (6 points) A professor separated her students' lab reports into two piles—those of the passing students and those of the failing students. Twenty-five students passed, and their average score was 78.35. Six students failed, and each one of them scored 63.5. What was the average score of all the students?

$$\frac{25(78.35) + 6(63.5)}{25+6} = \frac{2339.75}{31}$$

$$\approx \boxed{75.5}$$