

Math 153 - Test 3a

November 17, 2016

Name key

Score _____

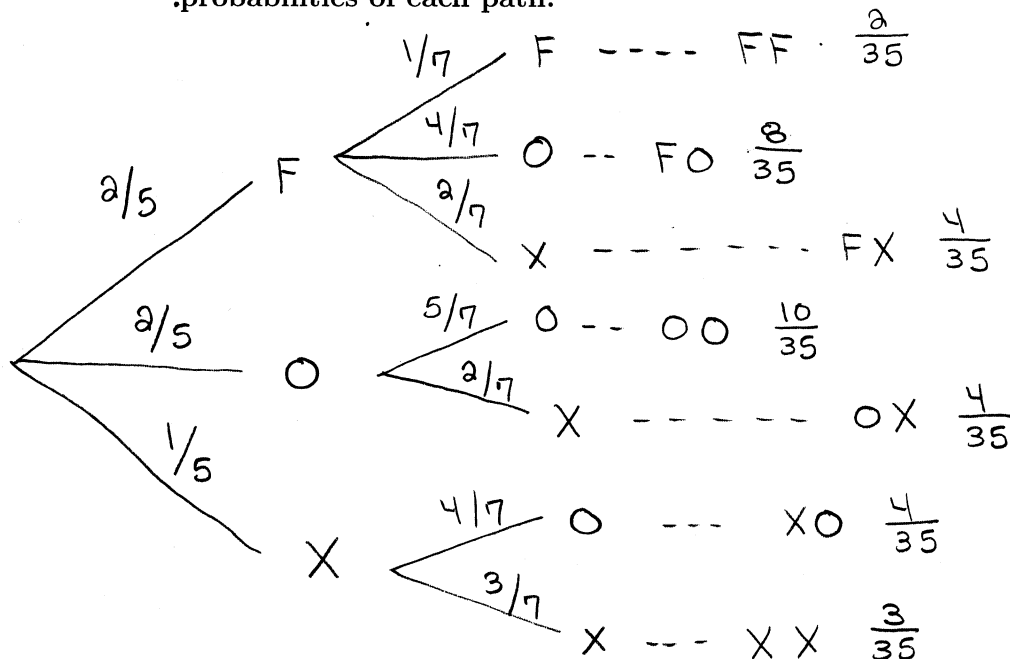
Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

F	F	O	O	X
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O	O	O	O	X	X
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- (a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**



- (b) What is the probability of selecting at least one X?

 $\{FX, OX, XO, XX\}$

$$\frac{4 + 4 + 4 + 3}{35} = \frac{15}{35}$$

- (c) What is the probability of selecting F from the first box or O from the second box?

 $\{FF, FO, FX, OO, XO\}$

$$\frac{2 + 8 + 4 + 10 + 4}{35} = \frac{28}{35}$$

2. (10 points) In studying the effectiveness of a test preparation course, the following data were collected.

	Passed Test	Failed Test	
Took Test-Prep Class	137	43	180
Did not take Test-Prep Class	213	105	318
	350	148	498

A person from this sample is selected at random.

- (a) What is the probability that the person passed the test?

$$\frac{350}{498} \approx 70.28\%$$

- (b) What is the probability that the person took the test preparation class and passed the test?

$$\frac{137}{498} \approx 27.51\%$$

- (c) What is the probability that the person passed the test given that he/she took the test preparation class?

$$\frac{137}{180} \approx 76.11\%$$

- (d) What is the probability that the person took the test preparation class given that he/she failed the test?

$$\frac{43}{148} \approx 29.05\%$$

- (e) Are taking the test preparation class and passing the test independent events? Give support for your answer!

$$(c): \text{Prob of passing given prep class} = \frac{137}{180} \approx 76.11\%$$

$$(a): \text{Prob of passing} = \frac{350}{498} \approx 70.28\%$$

2

Not equal \Rightarrow

Not independent

3. (6 points) Consider the table shown below.

x	$P(x)$
0	0.003
1	0.012
2	0.037
3	0.325
4	0.028
5	0.462
6	0.084
7	0.049

$$\mu = 4.33$$

$$\sigma = 1.3263...$$

(a) Does the table describe a probability distribution? Explain.

Yes, each $P(x)$ is between 0 & 1

$$\text{AND } \sum P(x) = 1$$

(b) What are unusually small values of x ?

$0 \text{ \& } 1$ BECAUSE $P(x \leq 1) = 0.003 + 0.012 = 0.015 < 0.05$

BUT $P(x \leq 2) = 0.052 > 0.05$

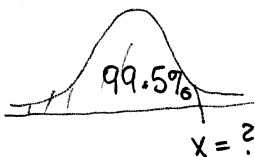
(c) What are unusually large values of x ?

7 BECAUSE $P(x \geq 7) = 0.049 < 0.05$

BUT $P(x \geq 6) = 0.133 > 0.05$

Using 5% Rule

4. (4 points) Heights of men are normally distributed with mean 69in and standard deviation 2.8in. A contractor wishes to determine the height of a basement ceiling that allows 99.5% of all men to stand without crouching. What would that height be?



$$\text{invNorm}(0.995, 69, 2.8) \approx 76.2 \text{ in}$$

5. (6 points) A candidate for a state senate seat found that in a certain region 68% of the registered voters recognized her name. Twenty-five registered voters are selected at random. Let x represent the number of voters in the sample who recognize the candidate's name.

(a) What is the mean value of x ?

BINOMIAL
 $n = 25$
 $p = 0.68$
 $q = 0.32$

$$np = 25 \times (0.68) = 17$$

(b) What is the standard deviation in the values of x ?

$$\sqrt{npq} = \sqrt{25(0.68)(0.32)} \approx 2.33$$

(c) What are the cutoff values for the unusually small and unusually large values of x ?

$$\mu - 2\sigma \approx 17 - 2(2.33) = 12.34$$

$$\mu + 2\sigma \approx 17 + 2(2.33) = 21.66$$

6. (5 points) The American Automobile Association (AAA) reports that the response times for vehicle emergency calls are normally distributed with a mean of 25 minutes and a standard deviation of 4.5 minutes.

(a) What is the probability that a response time will be 18 minutes or less?

NORMAL
 $\mu = 25$
 $\sigma = 4.5$

$$P(x \leq 18) = \text{normalcdf}(-999999, 18, 25, 4.5)$$

$$\approx 0.0599$$

(b) In a sample of 120 calls, about how many will have response times exceeding 32 minutes?

$$120 \times P(x > 32)$$

$$= 120 \times \text{normalcdf}(32, 999999, 25, 4.5)$$

$$4 \approx 7.19$$

ABOUT 7

7. (8 points) A certain Twitter user receives 6 tweets per hour, on average.

(a) What is the probability that the user receives exactly 4 tweets in any given hour?

Poisson
 $\mu = 6$

$$P(x=4) = \text{poissonpdf}(6, 4) \approx 0.1339$$

(b) What is the probability that the user receives at least 7 tweets in any given hour?

$$P(x \geq 7) = 1 - P(x \leq 6) \\ = 1 - \text{poissoncdf}(6, 6) \approx 0.3937$$

(c) What would be an unusually small number of tweets received in any given hour?

$$\mu - 2\sigma = \mu - 2\sqrt{\mu} = 6 - 2\sqrt{6} \\ \approx 1.10 \Rightarrow 1 \text{ or fewer}$$

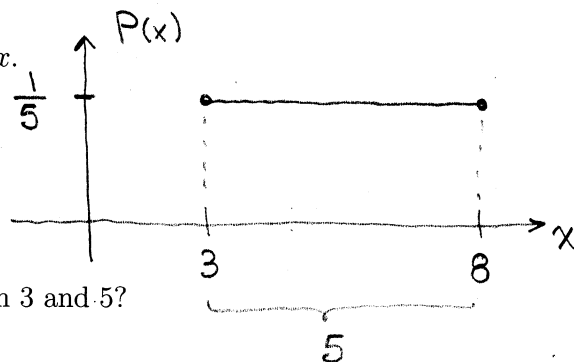
(d) What is the probability that the user receives exactly 150 tweets in any given day?

$\mu = 6 \cdot 24 \\ = 144$

$$P(x=150) = \text{poissonpdf}(144, 150) \\ \approx 0.0288$$

8. (6 points) A random number generator generates uniformly distributed random numbers between 3 and 8. Let the random variable x represent a randomly generated number.

(a) Sketch the probability density curve for x .



(b) What is the probability that x is between 3 and 5?

$$P(3 \leq x \leq 5) = (5-3) \left(\frac{1}{5} \right) \\ = \frac{2}{5} = 0.4$$

9. (4 points) In a certain heavily polluted lake, 62% of the frogs have unusual deformities. In a random sample of 30 captured frogs, what is the probability that more than 15 have unusual deformities?

BINOMIAL
 $p = 0.62$
 $n = 30$

$$P(x > 15) = 1 - P(x \leq 15)$$

$$= 1 - \text{binomcdf}(30, 0.62, 15)$$

$$\approx 0.8774$$

10. (7 points) Six \$1 bills, nine \$5 bills, five \$10 bills, and two \$20 bills are placed into a box, and one bill is selected at random. Let x be the value of the selected bill.

(a) Construct the probability distribution for the random variable x .

$$6 + 9 + 5 + 2 = 22$$

x	$P(x)$
1	$\frac{6}{22}$
5	$\frac{9}{22}$
10	$\frac{5}{22}$
20	$\frac{2}{22}$

(b) Find the expected value of x .

$$\mu = 1\left(\frac{6}{22}\right) + 5\left(\frac{9}{22}\right) + 10\left(\frac{5}{22}\right) + 20\left(\frac{2}{22}\right)$$

$$= \frac{141}{22} \approx \$6.41$$

(c) Find the standard deviation in the values of x .

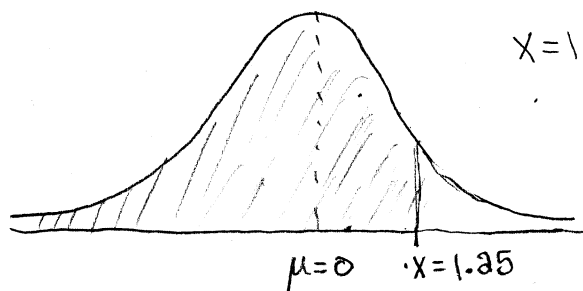
$$\sigma^2 = (1)^2\left(\frac{6}{22}\right) + (5)^2\left(\frac{9}{22}\right) + (10)^2\left(\frac{5}{22}\right) + (20)^2\left(\frac{2}{22}\right)$$

$$= \frac{1531}{22} - \left(\frac{141}{22}\right)^2$$

$$= 28.51446...$$

$$\Rightarrow \sigma \approx \$5.34$$

11. (4 points) Assume x is a random variable in the **standard normal distribution**. Sketch the standard normal density curve and shade the region under the curve corresponding to the probability $P(x < 1.25)$. Then compute the probability.



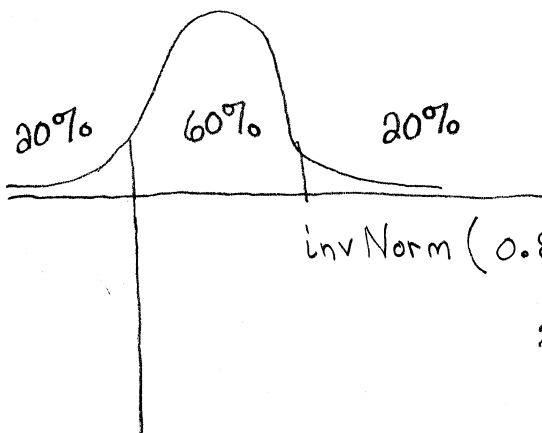
$x = 1.25$ IS A BIT PAST

THE INFLECTION PT RIGHT
OF CENTER

$$P(x < 1.25) = \text{normalcdf}(-999999, 1.25)$$

$$\approx 0.8944$$

12. (6 points) In a certain city, an automobile dealer finds that used car prices are normally distributed with mean \$8256 and standard deviation \$1150. The dealer decides to sell cars that appeal to the middle 60% of the market in terms of price. Find the minimum and maximum prices of the cars the dealer will sell.



$$\text{invNorm}(0.80, 8256, 1150)$$

$$\approx \$9223.86 = \text{MAX PRICE}$$

$$\text{invNorm}(0.20, 8256, 1150)$$

$$\approx \$7288.14 = \text{MIN PRICE}$$

Math 153 - Test 3b

November 17, 2016

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Score _____

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Tuesday, November 22. YOU MUST WORK INDIVIDUALLY ON THIS TEST—YOU WILL NOT BE GIVEN ANY CREDIT FOR GROUP WORK.

1. (8 points) The mean yearly Medicare Hospital Insurance benefit per person is \$4064. Suppose the benefits are normally distributed with standard deviation \$460. A random sample of 20 patients is obtained.

- (a) About how many patients in the sample of 20 have a benefit between \$3800 and \$4200?

$$\mu = 4064$$

$$\sigma = 460$$

$$20 \cdot P(3800 < x < 4200)$$

$$= 20 \text{ normalcdf}(3800, 4200, 4064, 460)$$

$$\approx 6.66 \Rightarrow \text{About } 7$$

- (b) What is the probability that the sample mean is less than \$3800.

$$\mu = 4064$$

$$\sigma = 460/\sqrt{20}$$

$$P(\bar{x} < 3800)$$

$$= \text{normalcdf}(-999999, 3800, 4064, 460/\sqrt{20})$$

$$\approx 0.0051$$

- (c) What is the probability that the sample mean is more than \$4100.

$$\mu = 4064$$

$$\sigma = 460/\sqrt{20}$$

$$P(\bar{x} > 4100)$$

$$= \text{normalcdf}(4100, 999999, 4064, 460/\sqrt{20})$$

$$\approx 0.3632$$

- (d) What sample mean is at the 90th percentile?

$$\text{Inv Norm}(0.90, 4064, 460/\sqrt{20})$$

$$\approx 4195.82$$

2. (10 points) The ages, in years, of the four U.S. presidents when they were assassinated in office are 56 (Lincoln), 49 (Garfield), 58 (McKinley), and 46 (Kennedy).

(a) Two of the ages are selected at random with replacement. List all possible samples and find the median of each sample.

$\{56, 56\} - 56$	$\{49, 56\} - 52.5$	$\{58, 56\} - 57$	$\{46, 56\} - 51$
$\{56, 49\} - 52.5$	$\{49, 49\} - 49$	$\{58, 49\} - 53.5$	$\{46, 49\} - 47.5$
$\{56, 58\} - 57$	$\{49, 58\} - 53.5$	$\{58, 58\} - 58$	$\{46, 58\} - 52$
$\{56, 46\} - 51$	$\{49, 46\} - 47.5$	$\{58, 46\} - 52$	$\{46, 46\} - 46$

(b) Summarize the sampling distribution in a probability distribution table.

X	46	47.5	49	51	52	52.5	53.5	56	57	58
$P(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

(c) Find the mean of the sample medians.

$$46\left(\frac{1}{16}\right) + 47.5\left(\frac{2}{16}\right) + \dots + 58\left(\frac{1}{16}\right) = \frac{836}{16} = 52.25$$

(d) Find the population median.

46, 49, 56, 58

$$\rightarrow \frac{49 + 56}{2} = 52.5$$

(e) Do the sample medians target the population median? Explain.

$(c) \neq (d) \Rightarrow$ Sample medians
DO NOT TARGET
Pop. median.

3. (6 points) In the problem above, the sample medians are the same as the sample means (because the sample size is two!). Find the population mean. Do the sample means target the population mean? Did you expect this?

$$\frac{46 + 49 + 56 + 58}{4} = 52.25$$

THIS IS EQUAL TO THE MEAN
OF THE SAMPLING DISTRIBUTION.

⇒ YES, SAMPLE MEANS
TARGET POP MEAN.

YES, I EXPECTED THIS.

AS DISCUSSED, THE MEAN IS
AN UNBIASED ESTIMATOR.