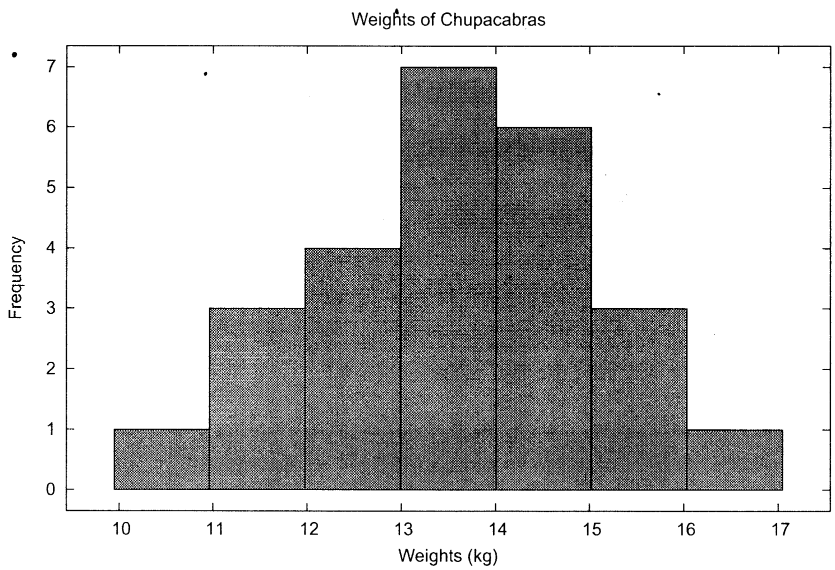
Math 153 - Final Exam

December 13, 2016

Name key Score

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) An eccentric animal breeder claims he has captured and is raising a number of chupacabras. The histogram below shows the weights of his chupacabras in kilograms.



(a) How many chupacabras does the breeder have in captivity?

$1+3+4+7+6+3+1 = 25$

(b) If the histogram was changed to a relative frequency histogram, what would be the height of the fourth bar (counting from the left)?

$\frac{7}{25} = 28\%$

(c) Are the weights of the chupacabras normally distributed? Explain.

THE WEIGHTS LOOK APPROXIMATELY NORMAL. THE OUTLINE OF THE HISTOGRAM HAS A ROUGHLY SYMMETRIC BELL SHAPE.

2. (6 points) Refer to Problem 1. Use class midpoints to compute the mean weight of the captive chupacabras.

$$\bar{x} \approx \left[1(10.5) + 3(11.5) + 4(12.5) + 7(13.5) + 6(14.5) + 3(15.5) + 1(16.5)\right] \div 25 = 13.58$$

3. (12 points) In a large sample of men, the mean height was 70.2 in with a standard deviation of 2.8 in. The mean weight was 172 lbs with a standard deviation of 29 lbs.

(a) Compute z-scores to determine which is "relatively" greater, 76.1 in or 232 lbs.

HEIGHT: $z = \dfrac{76.1 - 70.2}{2.8} \approx 2.107$ THE HEIGHT IS GREATER WEIGHT: $z = \dfrac{232 - 172}{29} \approx 2.069$

(b) Compute the coefficient of variation for the heights and the weights. Is there more spread in the heights or weights?

HEIGHTS: $\dfrac{2.8}{70.2} \approx 4.0\%$ WEIGHTS: $\dfrac{29}{172} \approx 16.9\%$

MUCH MORE SPREAD IN WEIGHTS!

(c) Based on the sample data, at what weight should a man be considered unusually light?

$172 - 2(29) = \boxed{114 \text{ lbs OR LESS}}$

4. (16 points) The numbers of tornadoes in Illinois for each year from 1965 to 1974 are shown below.

28, 11, 40, 8, 10, 17, 16, 30, 63, 107

(a) Find the range and the sample standard deviation.

RANGE $= 107 - 8 = 99$

$S \approx 30.99$

(b) Find the median, quartiles, and the interquartile range.

$Q_1 = 11$ $Q_3 = 40$

MED $= 22.5$ IQR $= 40 - 11 = 29$

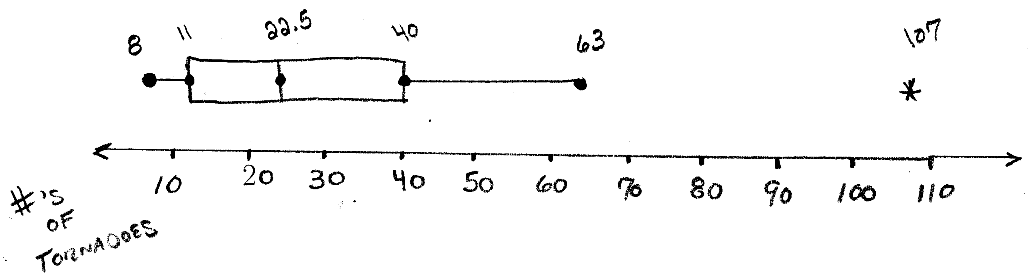
(c) Compute the cut-off values for outliers.

$11 - 1.5(29) = -32.5$

$40 + 1.5(29) = 83.5$

$\Rightarrow$ 107 IS THE ONLY OUTLIER.

(d) Sketch the modified boxplot.



5. (10 points) A number is selected at random from the first box and placed into the second box. Then a number is selected at random from the second box. The outcomes are recorded as ordered pairs of numbers such as $(0, 1)$.

0	1	1	1		1	1	1	2	2

(a) Find the sample space for this experiment.

$\{(0,0), (0,1), (0,2), (1,1), (1,2)\}$

(b) Is each outcome in your sample space equally likely? Explain.

No $(0,0)$ IS FAR LESS LIKELY THAN $(1,1)$

BECAUSE OF THE DIFFERING NUMBERS OF EACH.

(c) Let X be the event of drawing the number 0 from the first box. What is $\overline{X}$?

$X = \{(0,0), (0,1), (0,2)\}$

$\Rightarrow \boxed{\overline{X} = \{(1,1), (1,2)\}}$

(d) Let Y be the event of drawing the number 2 from the second box. What is $X \cup Y$?

$\boxed{X \cup Y = \{(0,0), (0,1), (0,2), (1,2)\}}$

(e) What is $X \cap Y$?

$\boxed{X \cap Y = \{(0,2)\}}$

6. (10 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

(a) Compute $P(A)$. $= 1 - 0.52 = 0.48$

(b) Compute $P(A \cap B)$. $= P(A) + P(B) - P(A \cup B)$

$= 0.48 + 0.55 - 0.766 = 0.264$

(c) Compute $P(B|A)$. $= \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.264}{0.48} = 0.55$

(d) Are A and B independent? Explain.

Yes, $P(B) = P(B|A)$

(e) What are the odds in favor of A?

$\dfrac{48}{52} = \dfrac{12}{13}$

7. (10 points) A certain sales firm receives, on average, 98 calls per day.

(a) On any given day, what is the probability of the firm receiving 103 or more calls?

POISSON $\mu = 98$

$P(x \geq 103) = 1 - P(x < 103)$

$= 1 - P(x \leq 102) = 1 - \text{poissoncdf}(98, 102) \approx 0.3199$

(b) On Tuesday the firm received 117 calls. Is that an unusually large number of calls?

$\mu + 2\sqrt{\mu} \approx 117.8$

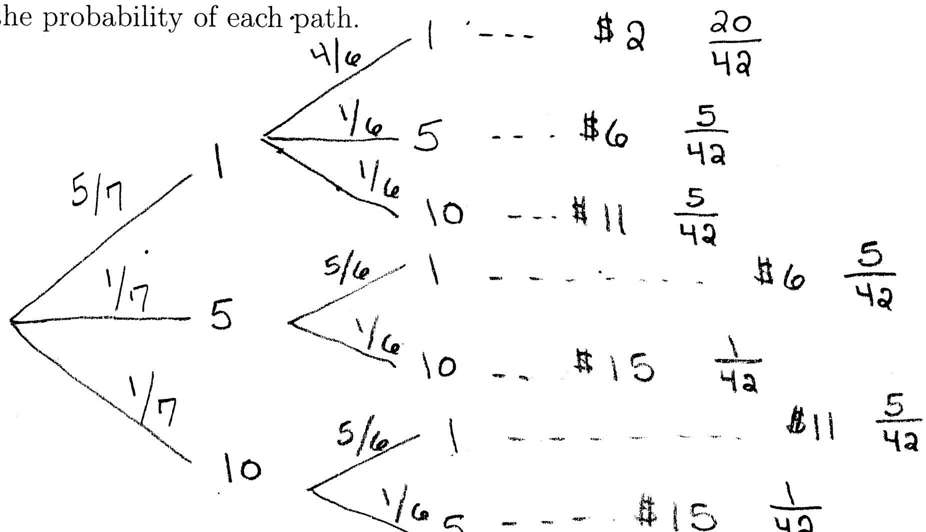
IT IS NOT UNUSUAL, BUT VERY CLOSE!

(c) On any given day, what would be an unusually small number of calls?

$\mu - 2\sqrt{\mu} \approx 78.2$ → 78 OR FEWER

8. A box contains five \$1-bills, one \$5-bill, and one \$10-bill. Two bills are selected at random without replacement. Let the random variable x represent the **total amount of money selected**.

(a) (6 points) Sketch the probability tree associated with the selection of the two bills. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

2, 6, 11, 15

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	P(x)
2	20/42
6	10/42
11	10/42
15	2/42

(d) (4 points) Find the mean value of x.

$$\mu = 2\left(\frac{20}{42}\right) + 6\left(\frac{10}{42}\right) + 11\left(\frac{10}{42}\right) + 15\left(\frac{2}{42}\right) = \frac{240}{42} \approx \$5.71$$

(e) (4 points) Find the standard deviation in the values of x.

$$\sigma = \sqrt{4\left(\frac{20}{42}\right) + 36\left(\frac{10}{42}\right) + 121\left(\frac{10}{42}\right) + 225\left(\frac{2}{42}\right) - \left(\frac{240}{42}\right)^2}$$

$$\approx \$4.16$$

9. (10 points) According to a recent Gallup poll, 24% of Americans view Japan as an economic threat. A random sample of 25 Americans is selected.

BINOMIAL
$n = 25$
$p = 0.24$
$q = 0.76$

(a) What is the probability that at least 8 of those sampled see Japan as a threat?

$$P(x \geq 8) = 1 - P(x < 8) = 1 - P(x \leq 7)$$
$$= 1 - \text{binomcdf}(25, 0.24, 7)$$
$$\approx 0.2349$$

(b) Suppose that only two people in the sample actually view Japan as a threat. Is that an unusually small number?

$$\mu - 2\sigma = np - 2\sqrt{npq} \approx 1.73$$

NO, 2 IS NOT UNUSUALLY SMALL.

(c) What is the probability that fewer than 4 of those surveyed see Japan as a threat?

$$P(x \leq 3) = \text{binomcdf}(25, 0.24, 3) \approx 0.1166$$

10. (10 points) The mean yearly Medicare spending per beneficiary is \$5694. Suppose the spendings are normally distributed with standard deviation \$612. A random sample of 15 patients is obtained.

NORMAL
CLT

$\mu_{\bar{x}} = \mu$
$= 5694$

(a) What is the standard deviation of the sampling distribution?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{612}{\sqrt{15}} \approx 158.02$$

(b) What is the probability that the sample mean is less than \$4800?

$$P(\bar{x} < 4800) = \text{normalcdf}(-99999, 4800, 5694, 612/\sqrt{15})$$
$$\approx 7.697 \times 10^{-9} \approx 0$$

(c) What is the probability that the sample mean is more than \$6700?

$$P(\bar{x} > 6700) = \text{normalcdf}(6700, 999999, 5694, 612/\sqrt{15})$$
$$\approx 9.721 \times 10^{-11} \approx 0$$

11. (12 points) The monthly amounts of paper waste generated by an American household are normally distributed with mean 28 lbs and standard deviation 2 lbs.

(a) What amount of paper waste is at the 75th percentile?

invNorm (0.75, 28, 2) ≈ 29.35 lbs

(b) What is an unusually large amount of paper waste?

28 + 2(2) = 32 lbs OR MORE

(c) In a sample of 500 households, about how many would generate more than 31.5 lbs of paper waste?

500 x normalcdf (31.5, 999999, 28, 2)

≈ 20.03 ⟹ ABOUT 20

12. (12 points) A survey of 35 adults found that the mean age of a person's primary vehicle is 5.6 years. Assume that the standard deviation of the population is 0.8 years.

(a) Construct a 95% confidence interval estimate for the mean age of all primary vehicles. State your conclusion in a complete sentence.

ZInterval w/ Stats

$\sigma = 0.8$ $n = 35$

$\bar{X} = 5.6$ C-Level = 0.95

(5.335, 5.865)

WE ARE 95% CONFIDENT THAT THE MEAN AGE OF A PERSON'S PRIMARY VEHICLE IS BETWEEN 5.335 yrs AND 5.865 yrs.

(b) Determine the sample size required to have a margin of error of ± 0.02 at the level $\alpha = 0.01$.

$\alpha = 0.01$

$\alpha/2 = 0.005$

$z_{\alpha/2} =$ invNorm (0.995)

≈ 2.576

$$n = \left(\frac{(2.576)(0.8)}{0.02} \right)^2$$

$= 10617.24$

Use $n = 10618$

13. (16 points) A large university reports that the mean salary of parents of an entering class is \$91,600. The university president randomly selects 28 families, and she finds the mean salary to be \$88,500 with a sample standard deviation of of \$9,915. Use the president's sample to test the university's reported claim at the level $\alpha = 0.10$.

(a) State the null and alternative hypotheses.

CLAIM: $\mu = 91600$

COUNTER: $\mu \neq 91600$

$$\boxed{\begin{aligned} H_0 &: \mu = 91600 \\ H_1 &: \mu \neq 91600 \end{aligned}}$$

(b) In order to test the claim, will you use a t-test or a z-test? Why? What assumption(s) must you make if your test is to be valid?

T-Test BECAUSE σ IS NOT KNOWN.

WE MUST ASSUME THE SALARIES ARE NORMALLY DISTRIBUTED ($n = 28 < 30$).

(c) Compute the test statistic.

T-Test w/ Stats

$t = -1.654$

$\mu_0 = 91600$ $n = 28$

$\bar{X} = 88500$ TWO-TAILED

$S_x = 9915$

(d) Find the P-value and draw a conclusion about the university's claim.

P-VALUE $= 0.1096 > \alpha$

SINCE P-VALUE IS NOT $< \alpha$, WE DO NOT REJECT H_0.

AT THE LEVEL $\alpha = 0.10$, THERE IS NOT SUFFICIENT EVIDENCE TO REJECT THE CLAIM THAT $\mu = \$91600$.