Math 153 - Test 2
October 12, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) A collection of numbers has mean 32.5 and standard deviation 8.3.

   (a) Give an example of a z-score of an unusually small number that might be in this data set.

   -2.5 would represent a raw score 2.5 std. dev's below the mean

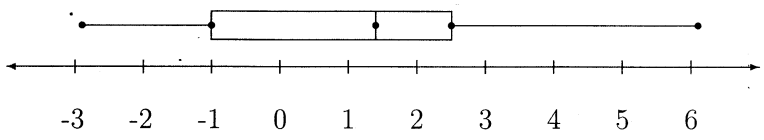
   (b) How do you know your z-score above corresponds to an unusually small number?

   Any z-score less than -2 would represent an unusually small number.

   (c) What data value in the collection would have the z-score that you gave above?

   $32.5 - 2.5(8.3) = 11.75$

2. (8 points) The boxplot shown below describes a certain collection of data. Find approximate values for the median, first and third quartiles, and the interquartile range. Based on your approximations, what would be the cutoff values for outliers?



Median ≈ 1.3

$Q_1 \approx -1$

$Q_3 \approx 2.5$

$IQR \approx 2.5 - (-1) = 3.5$

Cutoffs:

$Q_1 - 1.5(IQR) \approx -1 - 1.5(3.5) = -6.25$

$Q_3 + 1.5(IQR) \approx 2.5 + 1.5(3.5) = 7.75$

3. (9 points) Forty-five full-time PSC students were selected at random and asked how many hours per week they normally spend studying outside of class. The results are shown below.

0	0	0	1	1.5	2	2	2	2
2	2	2.5	3	3	3.5	3.5	4	4
4.5	5	5	6	6	6	6	7	7
7	8	8	8	8	9	9	10	10
10	10	12	14	14	15	20	20	25

(a) Find the percentile for the value 10.

$$\frac{\text{NUMBER OF VALUES} < 10}{45} = \frac{34}{45} = 0.7\overline{5}$$

$\Rightarrow$ 76TH PERCENTILE

(b) Find the value of the 90th percentile.

$$\frac{L}{45} = 0.90 \Rightarrow L = 40.5$$

$\Rightarrow$ 41ST VALUE IS 14

(c) Find the value of the 40th percentile.

$$\frac{L}{45} = 0.40 \Rightarrow L = 18$$

$\Rightarrow$ $\frac{18^{TH} + 19^{TH}}{2} = \frac{4+4.5}{2} = 4.25$

4. (4 points) A standard six-sided die is rolled. Let A be the event of rolling a 5 or 6.

(a) Determine $P(\bar{A})$.

$$P(A) = \frac{2}{6} \Rightarrow P(\bar{A}) = \frac{4}{6}$$

(b) Determine the odds in favor of A.

$$P(A) = \frac{2}{6} \Rightarrow \text{ODDS ARE } \frac{2}{4} \text{ OR } 1:2$$

5. (10 points) The upper leg lengths, in centimeters, of 20 adult females are shown below (in numerical order).

27.2	31.1	33.2	36.0	36.3	38.1	38.2	38.2	38.7	39.0
39.2	39.7	39.9	40.2	40.3	41.3	41.6	41.6	43.8	48.5

Determine the quartiles, the interquartile range, and the cutoff values for outliers. Then sketch the modified boxplot. (Do all work by hand, but you may check your work on your calculator.)

$Q_1 = \frac{36.3 + 38.1}{2} = 37.2$

$\text{MED} = \frac{39.0 + 39.2}{2} = 39.1$

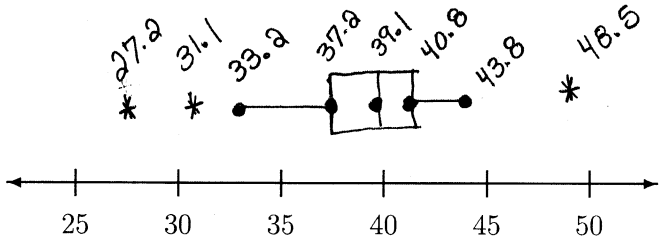
$Q_3 = \frac{40.3 + 41.3}{2} = 40.8$

$\text{IQR} = 40.8 - 37.2 = 3.6$

CUTOFFS:

$37.2 - 1.5(3.6) = 31.8$

$40.8 + 1.5(3.6) = 46.2$



6. (6 points) A letter is selected at random from the word *racecars*. Let A be the event of selecting a vowel, and let B be the event of selecting the letter e.

(a) Determine $P(A|B)$.

vowel given "e"

100%

(b) Determine $P(B|A)$.

"e" given a,e,a

$\frac{1}{3} \approx 33.3\%$

(c) Determine $P(B|\bar{A})$.

"e" given r,c,c,r,s

$0 = 0\%$

7. (12 points) The numbers of students at a certain college are described in the table below.

	Female	Male	
Part-time	1764	1001	2765
Full-time	1502	1463	2965
	3266	2464	5730

A college student is selected at random.

(a) What is the probability that the student is a female?

$$\frac{3266}{5730} \approx 57\%$$

(b) What is the probability that the student is a full-time student?

$$\frac{2965}{5730} \approx 51.7\%$$

(c) What is the probability that the student is a female or a full-time student?

$$\frac{1764 + 1502 + 1463}{5730} = \frac{4729}{5730} \approx 82.5\%$$

(d) What is the probability that the student is a female, full-time student?

$$\frac{1502}{5730} \approx 26.2\%$$

(e) What is the probability that the student is a full-time student given that she is a female?

$$\frac{1502}{3266} \approx 46\%$$

(f) Are being a female student and being a full-time student independent events? Use probabilities to support your answer.

No PROB OF FULL-TIME GIVEN FEMALE $\approx 46\%$ (e)

IS NOT EQUAL TO

PROB OF FULL-TIME $\approx 51.7\%$ (b)

8. (5 points) Suppose that A, B, C, and D are disjoint (mutually exclusive) events that exhaust the sample space? If $P(B) = 3/5$ and $P(C) = 1/5$, then what is the probability of $A \cup D$? (Show work or explain.)

$P(A \cup D) = P(A) + P(D)$

$P(A) + P(B) + P(C) + P(D) = 1$

$\frac{3}{5} + \frac{1}{5}$

$\frac{4}{5}$

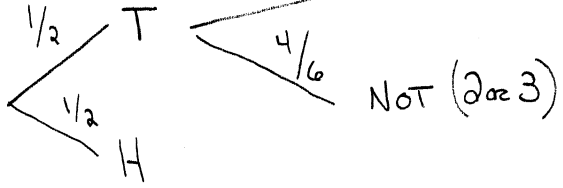
$P(A \cup D) = \frac{1}{5}$

9. (4 points) The odds in favor of the event Z are 3 : 7. What are the odds against Z? What is the probability of Z?

$\frac{3}{10}$

$\frac{7}{3}$

10. (4 points) A coin is flipped and a standard die is rolled. What is the probability of getting tails on the flip **and** getting 2 or 3 on the roll? (This problem is asking about a single event.)



$\frac{1}{2}\left(\frac{2}{6}\right) = \frac{1}{6}$

11. (4 points) A jar contains 4 quarters, 12 dimes, and 18 pennies. One coin is selected at random. What is the best sample space for this experiment? Are your outcomes equally likely?

$\{Q, D, P\}$ or $\{25, 10, 1\}$

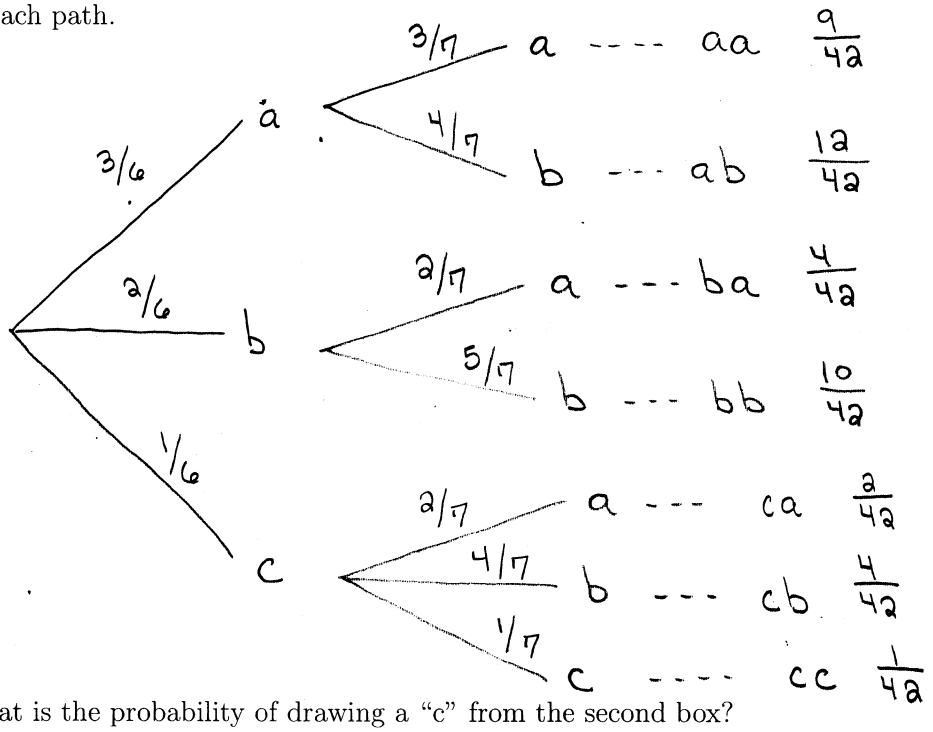
Outcomes are not equally likely because there are different numbers of each coin.

12. (12 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

a	a	a	b	b	c

a	a	b	b	b	b

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) What is the probability of drawing a "c" from the second box?

cc ---- $P(\{cc\}) = \frac{1}{42}$

(c) What is the probability of drawing a "c" or a "b" (from either box)?

ANYTHING BUT aa

HAS PROB $1 - \frac{9}{42} = \frac{33}{42}$

13. (6 points) Indicate whether each statement is true or false. (You may get partial credit for incorrect answers if you show work or explain.)

(a) TRUE If A and B are disjoint (mutually exclusive), then $P(A \cup B) = P(A) + P(B)$.

(b) TRUE If C is a certain event, then $\overline{C}$ is an impossible event.

(c) FALSE If $P(X) = 0.4$, then the odds in favor of X are ~~4 : 10.~~ 4 : 6

14. (10 points) Suppose A and B are events such that $P(A) = 0.54$, $P(\overline{B}) = 0.32$, and $P(A \cup B) = 0.66$.

(a) Compute $P(B)$.

$1 - 0.32 = \boxed{0.68}$

(b) Compute $P(A \cap B)$.

$0.66 = 0.54 + 0.68 - P(A \cap B)$

$\Rightarrow P(A \cap B) = \boxed{0.56}$

(c) Compute $P(A|B)$.

$$\frac{P(A \cap B)}{P(B)} = \frac{0.56}{0.68} \approx \boxed{0.82}$$

(d) Are A and B independent? Explain.

No, BECAUSE $P(A|B) \approx 0.82 \neq 0.54 = P(A)$

(e) What are the odds against A?

$P(A) = 0.54 \Rightarrow$ ODDS FOR A ARE $\frac{54}{46}$

$\Rightarrow$ ODDS AGAINST ARE

$$\frac{46}{54} = \boxed{\frac{23}{27}}$$