Math 153 - Test 3
November 16, 2017

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The values of the random variable x and its probabilities are shown below.

x	0	1	2	3	4	5	6
$P(x)$	0.03	?	0.41	0.21	0.29	?	0.01

Find possible values for $P(1)$ and $P(5)$ so that the table describes a probability distribution **and** both $x = 1$ and $x = 5$ are unusual.

$0.03 + 0.41 + 0.21 + 0.29 + 0.01 = 0.95$

$P(1) + P(5) = 0.5$

- AND -

$P(1) < 0.02$

$P(5) < 0.04$

LET $P(1) = 0.015$

$P(5) = 0.035$

2. (9 points) Suppose the heights of trees in an orchard are normally distributed with mean 112 inches and standard deviation 14 inches.

(a) What percent of trees in the orchard have heights less than 85 inches?

$P(x < 85) = \text{normalcdf}(-999999, 85, 112, 14)$

$\approx 0.0269 = 2.69\%$

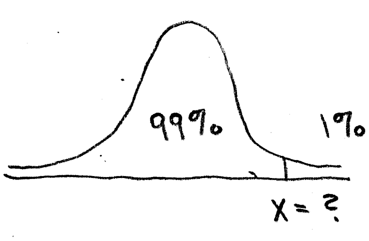
(b) What is the probability that a randomly selected tree is between 110 and 113 inches tall?

$P(110 \le x \le 113) = \text{normalcdf}(110, 113, 112, 14)$

$\approx 0.0853 = 8.53\%$

(c) A scientist looking for a certain inherited trait wants to study the tallest 1% of trees in the orchard. How tall are those trees?

$\text{invNorm}(0.99, 112, 14) \approx 144.57 \text{ in}$



144.57 in OR TALLER

3. (15 points) The probability distribution for the random variable x is shown below.

x	0	1	2	3	4	5
$P(x)$	0.12	0.03	0.74	0.07	0.01	0.03

(a) What two things about the table above show that it is a probability distribution?

① $0 \le P(x) \le 1$ FOR EACH X

② $\sum P(x) = 0.12 + 0.03 + 0.74 + 0.07 + 0.01 + 0.03 = 1$

(b) What is the mean value of x?

$\mu = 0(0.12) + 1(0.03) + 2(0.74) + 3(0.07) + 4(0.01) + 5(0.03)$

$= 1.91$

(c) What is the standard deviation in x?

$\sigma^2 = 0(0.12) + 1(0.03) + 4(0.74) + 9(0.07) + 16(0.01) + 25(0.03) - 1.91^2$

$= 0.8819$

$\sigma = \sqrt{0.8819} \approx 0.939$

(d) Determine $P(x > 2)$.

$P(3) + P(4) + P(5) = 0.07 + 0.01 + 0.03$

$= 0.11$

(e) Determine all unusual values of x.

Using CUMULATIVE 5% RULE:

NO VALUES ARE UNUSUALLY SMALL.

4 & 5 ARE UNUSUALLY LARGE.

Using 2 STD. DEVS:

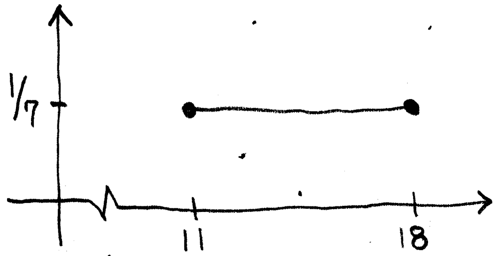
$\mu + 2\sigma \approx 3.788$

$\mu - 2\sigma \approx 0.032$

$\Rightarrow$ 0, 4, 5 ARE UNUSUAL.

4. (12 points) Suppose the travel times from your house to your favorite restaurant are uniformly distributed between 11 minutes and 18 minutes.

   (a) Sketch the graph of the probability density function.



$18 - 11 = 7$

HEIGHT $= \frac{1}{7}$

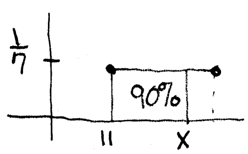
   (b) What is the probability that your travel time is exactly 12 minutes?

$$P(x=12) = \boxed{0}$$

   (c) What is the probability that your travel time is between 11 and 13 minutes?

$$P(11 \le x \le 13) = \frac{1}{7}(13-11) = \boxed{\frac{2}{7}}$$

   (d) What travel time is at the 90th percentile?



$$\frac{1}{7}(x-11) = 0.90$$

$$x - 11 = 6.3 \Rightarrow \boxed{x = 17.3 \text{ MIN}}$$

5. (9 points) In 2011, the mean age of U.S. college students was 25.0 years, with a standard deviation of 9.5 years. A simple random sample of size 15 is obtained.

   (a) Explain why the Central Limit theorem does not tell us anything about the distribution of sample means.

THE AGES OF COLLEGE STUDENTS ARE NOT SAID TO BE NORMALLY DISTRIBUTED, AND THE SAMPLE SIZE IS TOO SMALL.

   (b) Change something about the problem situation so that the Central Limit theorem applies.

CHANGE TO SAMPLES OF SIZE 64.

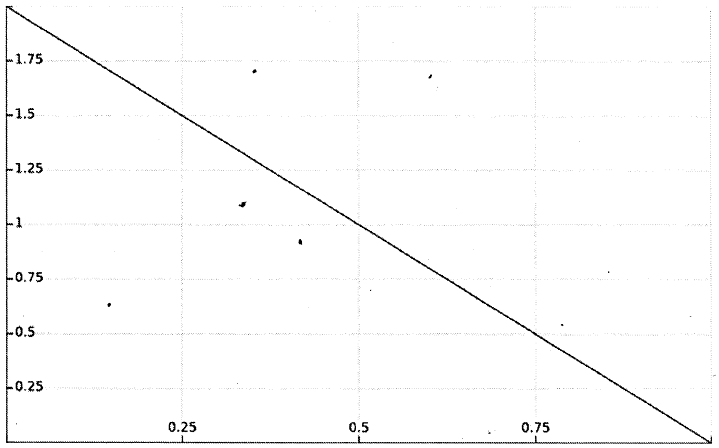
NOW $\mu_{\bar{x}} = 25.0$ AND $\sigma_{\bar{x}} = 9.5/\sqrt{64}$ AND CLT APPLIES.

   (c) With your change from part (b) in place, determine the probability that your sample mean is greater than 26.0 years.

$$P(\bar{x} > 26.0) = \text{normalcdf}(26, 999999, 25, 9.5/8)$$

$$\approx \boxed{0.1999 = 19.99\%}$$

6. (5 points) Is the graph shown below a probability density curve? Explain why or why not.



YES, THE GRAPH LIES ABOVE OR ON THE HORIZONTAL AXIS,

AND AREA UNDER GRAPH $= \frac{1}{2}(1)(2) = 1$.

7. (9 points) In a certain urban county, there are an average of 464 births per year.

Poisson

$\mu = \lambda = 464$

$\sigma = \sqrt{\lambda}$

(a) Recall that the population coefficient of variation (CV) is computed with the formula $CV = \sigma/\mu$. Find the CV for the births per year.

$$CV = \frac{\sigma}{\mu} = \frac{\sqrt{464}}{464} \approx 4.64\%$$

(b) In any given year, what is the probability that there are fewer than 450 births?

$$P(X < 450) = P(X \leq 449)$$

$$= \text{poissoncdf}(464, 449) \approx 0.2518 = 25.18\%$$

(c) In any given **month**, what is the probability that there are exactly 38 births?

$\lambda = \frac{464}{12}$

$$P(X = 38) = \text{poissonpdf}\left(\frac{464}{12}, 38\right)$$

$$\approx 0.0642 = 6.42\%$$

BINOMIAL
$n=35$
$p=0.71$
$q=0.29$

8. (12 points) 71% of Americans believe illegal drugs are a serious problem in the U.S. Suppose a simple random sample of 35 Americans is obtained.

(a) What is the probability that exactly 18 people believe drugs are a serious problem?

$$P(x=18) = \text{binompdf}(35, 0.71, 18) \approx 0.0069 = 0.69\%$$

(b) What is the probability that no fewer than 20 people believe drugs are a serious problem?

$$P(x \geq 20) = 1 - P(x < 20) = 1 - P(x \leq 19)$$

$$= 1 - \text{binomcdf}(35, 0.71, 19) \approx 0.9736 = 97.36\%$$

(c) A quick check will show you that $P(x = 29) = 0.04691$. Does this indicate the 29 is an unusual value? Explain.

No, you NEED TO CHECK A CDF, NOT A PDF.

IN FACT, $P(x \geq 29) = 0.0824$. 29 IS NOT UNUSUAL!

(d) In the sample of 35, what would be an usually small number of people who think drugs are a serious problem? (Be sure to show your work.)

$$np - 2\sqrt{npq} = 35(0.71) - 2\sqrt{35(0.71)(0.29)}$$

$$\approx 19.48 \Rightarrow$$ 19 OR FEWER ARE UNUSUAL.

9. (9 points) The Ford Edge has a mean highway gas mileage of 27 miles per gallon (mpg) with a standard deviation of 3 mpg. A rental car company buys a fleet of 60 of these cars.

(a) What is the probability that the mean gas mileage of the fleet exceeds 26.5 mpg?

$$P(\bar{x} > 26.5) = \text{normalcdf}(26.5, 9999999, 27, 3/\sqrt{60})$$

$$\approx 0.9016 = 90.16\%$$

(b) Would it be unusual if the mean gas mileage of the fleet were less than 26 mpg? Show work or explain.

$$P(\bar{x} < 26) = \text{normalcdf}(-999999, 26, 27, 3/\sqrt{60})$$

$$\approx 0.0049 < 0.05$$ YEP! THAT'S UNUSUAL.

(c) In order to answer the question in part (a), did you have to assume that the gas mileages were normally distributed? Explain.

No, SINCE $n = 60 > 30$, THE CLT APPLIES AND THE SAMPLING DIST. IS APPROX. NORMAL.

10. (4 points)

(a) What does it mean for an estimator to be biased?

THE ESTIMATOR DOES NOT TARGET THE POPULATION PARAMETER BEING ESTIMATED. IN OTHER WORDS, THE MEAN OF THE SAMPLING DISTRIBUTION IS NOT THE POP. PARAMETER.

(b) Give an example of a biased estimator.

RANGE, MEDIAN

(c) Give an example of an unbiased estimator.

MEAN, PROPORTION

11. (4 points) A bank has an average of 31.8 customers per hour, with most customers visiting between 11am and 2pm. Let x represent the number of customers in any given hour.

(a) What are the possible values of x?

$X = 0, 1, 2, 3, \ldots$

(b) Suppose you were asked to compute $P(x > 35)$. Why is it inappropriate to use the Poisson CDF?

REQUIRES OCCURRENCES TO BE SPREAD OUT EVENLY. NOT HAPPENING HERE.

12. (3 points) A fun size bag of Skittles contains 13 candies, 6 of which happen to be yellow. (Therefore the probability of randomly selecting a yellow candy is 6/13.) Suppose 10 candies are selected at random and eaten. Let x represent the number of yellow candies in the sample. Are the values of x in a binomial distribution? Explain.

NO, PROB OF YELLOW CHANGES WITH EACH CANDY SELECTED.

13. (3 points) An automobile dealer finds that used car prices are normally distributed with mean \$14,250 and standard deviation \$2080. The dealer selects random samples of 15 cars. Are the sample means normally distributed? Explain.

YES, EVEN THOUGH THE SAMPLES ARE SMALL, THE CLT APPLIES BECAUSE THE PRICES ARE NORMALLY DISTRIBUTED.