Math 153 - Final Exam

December 12, 2017

Name ___key___

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (20 points) The probability distribution for the random variable x is shown below.

x	0	1	2	3	4
$P(x)$	0.04	0.40	0.48	0.02	0.06

(a) Is the variable x a <u>discrete</u> or a continuous random variable? How do you know?

↳ X ONLY TAKES 5 DIFFERENT VALUES:

$$X = 0, 1, 2, 3, 4$$

(b) What two things about the table above show that it is a probability distribution?

• $0 \leq P(x) \leq 1$ FOR EACH X

• $\sum P(x) = 0.04 + 0.40 + 0.48 + 0.02 + 0.06 = 1$

(c) What is the mean value of x?

$$\mu = 0(0.04) + 1(0.40) + 2(0.48) + 3(0.02) + 4(0.06)$$
$$= 1.66$$

(d) What is the standard deviation in x?

$$\sigma^2 = 0(0.04) + 1(0.40) + 4(0.48) + 9(0.02) + 16(0.06) - (1.66)^2$$
$$= 0.7044 \implies \sigma = \sqrt{0.7044} \approx 0.84$$

(e) Use the mean and standard deviation to determine the unusual values of x.

$$\mu - 2\sigma \approx 1.66 - 2(0.84) = -0.02$$
$$\mu + 2\sigma \approx 1.66 + 2(0.84) = 3.34$$

X = 4 IS UNUSUAL

(f) Use the 5% rule to determine the unusual values of x.

BASED ON 5% RULE, ONLY X = 0

IS UNUSUAL.

2. (6 points) The mean of 32 test scores is 70.25. Two tests taken late had scores 58.5 and 67. Find the mean of all the test scores.

$$\bar{X} = \frac{32(70.25) + 58.5 + 67}{34}$$

$$= \frac{2373.5}{34}$$

$$\approx 69.81$$

3. (8 points) For each part of this problem, sketch a boxplot that would correspond to a data set with the given characteristics.
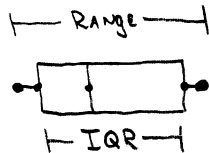
   (a) There is a single outlier in the upper half of the data.
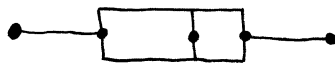
MUST BE MORE THAN
1.5 IQR's FROM Q_3

*

   (b) The IQR is a bit less than the range.

├── RANGE ──┤

├ IQR ┤

4. (8 points) For Yellowstone's Old Faithful geyser, the mean time between eruptions is 1.55 hr with a standard deviation of 0.11 hr. For Yellowstone's Lone Star geyser, the mean is 3.00 hr with a standard deviation of 0.16 hr. Compute the coefficient of variation (CV) for each geyser. Which geyser's eruption cycle has more variation?

OF: $\frac{0.11}{1.55} \approx 7.1\%$

OLD FAITHFUL HAS MORE VARIATION.

LS: $\frac{0.16}{3.00} \approx 5.3\%$

5. (6 points) Suppose that mortgage interest rates are normally distributed with mean 4.080 and standard deviation 0.164. Steve will not consider an interest rate offer from a bank unless it is in the lowest 25% of rates. What is the greatest interest rate that Steve will consider?

$$\text{inv Norm}\left(0.25,\ 4.080,\ 0.164\right)$$

$$\approx \boxed{3.969}$$

6. (12 points) A jar contains 4 quarters, 3 dimes, and 1 nickel. Two coins are selected at random (without replacement).

(a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**
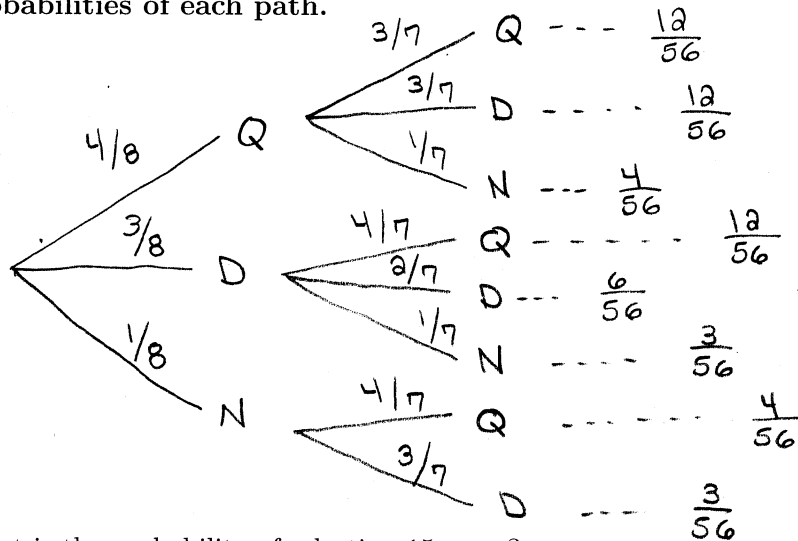
(b) What is the probability of selecting 15 cents?

DN or ND

$$\frac{3}{56} + \frac{3}{56} = \boxed{\frac{6}{56}}$$

(c) What is the probability of selecting more than 15 cents?

EVERYTHING EXCEPT DN or ND

$$1 - \frac{6}{56} = \boxed{\frac{50}{56}}$$

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7. (6 points)

   (a) What does it mean for an estimator to be biased?

   THE ESTIMATOR DOES NOT TARGET THE POPULATION PARAMETER
   BEING ESTIMATED. IN OTHER WORDS, THE MEAN OF THE
   SAMPLING DISTRIBUTION IS NOT THE POPULATION PARAMETER.

   (b) Give an example of a biased estimator.

   RANGE, MEDIAN

   (c) Give an example of an unbiased estimator.

   MEAN, PROPORTION

8. (12 points) A ski gondola carries a maximum of 12 people or 2004 lb. Assume that
   weights of men are normally distributed with mean 182.9 lb and standard deviation
   40.8 lb.

   (a) If a man is randomly selected, find the probability that his weight is greater than
       167 lb.

   $$P(x > 167) = \text{normalcdf}(167, 999999, 182.9, 40.8)$$

   $$\approx \boxed{0.6516}$$

   (b) If 12 different men are randomly selected, find the probability that their mean
       weight is greater than 167 lb.

   $$P(\bar{x} > 167) = \text{normalcdf}(167, 999999, 182.9, 40.8/\sqrt{12})$$

   $$\approx \boxed{0.9115}$$

   (c) For engineers designing a ski gondola, which probability is more relevant: the
       result from part (a) or the result from part (b)? Briefly explain why.

   WITH 12 PEOPLE ON A GONDOLA,
   INDIVIDUAL WEIGHTS DO NOT
   MATTER, BUT THE WEIGHT
   OF THE WHOLE GROUP IS
   VERY IMPORTANT.

4

9. (12 points) Suppose that 16% of M & M's milk chocolate candies are green. A random sample of 455 M & M's is obtained.

(a) About how many candies would you expect to be green?

$$\mu = (0.16)(455) = 72.8$$

About 73

(b) What is the probability that at least 80 candies in your sample are green?

$$P(x \geq 80) = 1 - P(x < 80) = 1 - P(x \leq 79)$$
$$= 1 - \text{binomcdf}(455, 0.16, 79) \approx 0.1947$$

(c) In the sample of 455 candies, what would be an unusually small number of green candies?

$$\sigma = \sqrt{(455)(0.16)(0.84)} \approx 7.82$$

$$\mu - 2\sigma \approx 57.16 \Rightarrow \boxed{57 \text{ OR FEWER}}$$

10. (12 points) As a manager for an advertising company, you must plan a campaign designed to increase Twitter usage. You want to first determine the percentage of adults who are familiar with Twitter. How many adults must you survey in order to be 90% confident that your estimate is within five percentage points of the true population percentage?

$\alpha = 0.10$

$\alpha/2 = 0.05$

$E = 0.05$

$Z_{\alpha/2} = $ invNorm(0.95) ≈ 1.645

(a) Assume that nothing is known about the percentage of adults familiar with Twitter.

$$n = \frac{[1.645]^2 (0.25)}{(0.05)^2} \approx 270.6$$

Use n = 271

(b) Assume that a recent survey suggests that about 85% of adults are familiar with Twitter.

$$n = \frac{[1.645]^2 (0.85)(0.15)}{(0.05)^2} \approx 138.007$$

Use n = 139

(c) Could you simply survey some adults at Prairie State College? Explain your reasoning.

No, THAT WOULD NOT BE A

SIMPLE RANDOM SAMPLE

OF ADULTS.

11. (20 points) A simple random sample of 10 employees is selected from a large tax preparation firm. Each of the employees is given the same tax information and timed while completing the tax return. Their times, in minutes, are given below:

$$15.0, \ 57.7, \ 25.0, \ 44.2, \ 34.9, \ 53.4, \ 37.3, \ 40.9, \ 55.0, \ 30.8$$

(a) Compute the sample mean and sample standard deviation.

FROM CALCULATOR...:

$$\bar{X} = 39.42, \quad S \approx 13.75$$

(b) Based on the mean and standard deviation, what is an unusually large completion time?

$$\bar{X} + 2\sigma \approx 66.92$$

MORE THAN 66.92 MINUTES

(c) Compute a 95% confidence interval estimate for the firm's overall mean completion time.

T Interval w/ Data gives

$$(29.586, 49.254)$$

(d) Give an interpretation of your confidence interval in a complete sentence.

WE ARE 95% CONFIDENT THAT THE TRUE MEAN COMPLETION TIME IS BETWEEN 29.6 MIN AND 49.3 MIN.

(e) What assumption must be made if we are to believe that the confidence interval estimate is accurate?

SINCE $n = 10 < 30$, WE MUST ASSUME THE DISTRIBUTION OF THE POPULATION OF COMPLETION TIMES IS NORMAL.

12. (12 points) You believe that at least 60% of adults disapprove of the proposed changes to the federal tax code. In a simple random sample of 1020 U.S. adults, a recent Gallup poll survey found that 56% disapproved of the proposed changes.

(a) State the null and alternative hypotheses.

CLAIM: $p \geq 0.60$

COUNTER: $p < 0.60$.

$H_0: p = 0.60$

$H_A: p < 0.60$

(b) Compute the test statistic and the P-value.

1-Prop Z Test

$P_0: 0.60$

$X = 571$

$n = 1020$

TEST STAT =

$Z = -2.62$

P-VALUE $= 0.00439$

(c) Draw a conclusion about your original claim at the level $\alpha = 0.01$. Write your conclusion in a complete sentence.

SINCE   P-VALUE $\approx 0.004 < \alpha = 0.01$,

WE REJECT THE NULL HYPOTHESIS.

THE EVIDENCE SUPPORTS THE NOTION THAT

THE PROPORTION IS LESS THAN 60%.

13. (16 points) A personnel manager collected the following data from a random sample of 8 employees. The data show ownership of company stock versus years with the company.

Years with company, x	Number of shares, y
6	300
12	408
14	560
6	252
9	288
13	650
15	630
9	522

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength (and direction) of the linear relationship between x and y.

Lin Reg...

$$r = 0.8486$$

THIS INDICATES THAT THERE IS A STRONG, POSITIVE LINEAR RELATIONSHIP.

(b) Find the regression equation.

$$\hat{y} = 38.76x + 44.31$$

(c) Use your regression equation to predict the number of shares owned by an employee who has been at the company for 10 years.

$$38.76(10) + 44.31 = 431.91$$

$\Rightarrow$ ABOUT 432 SHARES

(d) An employee has 450 shares of company stock. Predict how long the employee has been with the company.

$$450 = 38.76x + 44.31$$

$\Rightarrow$ $X = 10.47$ years