Math 153 - Test 2
March 20, 2014

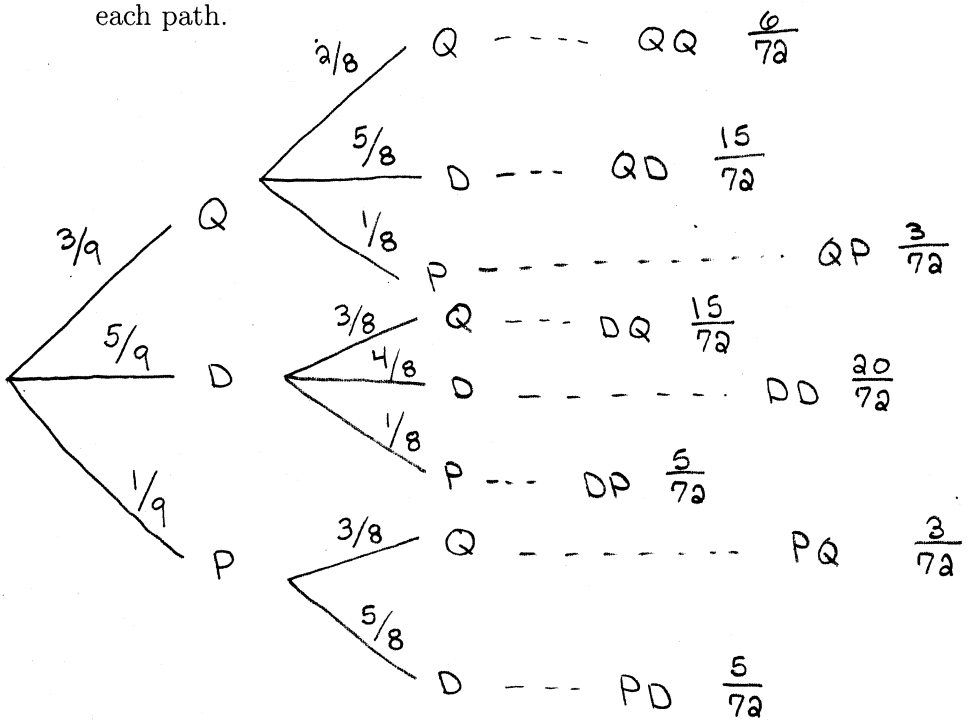
Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) A jar contains 3 quarters, 5 dimes, and 1 penny. Two coins are selected at random without replacement.

   (a) Sketch the complete tree diagram for the experiment. Include the probabilities of each path.



   (b) What is the probability of getting at least 30 cents? QQ, QD, DQ

$$\frac{6}{72} + \frac{15}{72} + \frac{15}{72} = \frac{36}{72} = \boxed{\frac{1}{2}}$$

   (c) What are the odds in favor of getting exactly 11 cents? DP, PD

PROB IS $\frac{5}{72} + \frac{5}{72} = \frac{10}{72}$

ODDS IN FAVOR ARE $\frac{10}{62} = \boxed{\frac{5}{31}}$

2. (10 points) The following table shows the numbers of chocolate chips counted in each of 24 Hannaford Chocolate Chip cookies.

11	12	12	12	13	13	13	13	14	14	14	14
14	14	15	15	15	15	16	16	16	16	17	21

Determine the quartiles, the interquartile range, and the cutoff values for outliers. Then sketch the modified boxplot.

$Q_1 = \frac{13+13}{2} = \boxed{13}$

$Q_2 = \frac{14+14}{2} = \boxed{14}$

$Q_3 = \frac{15+16}{2} = \boxed{15.5}$

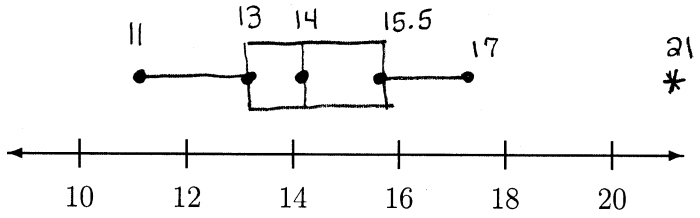
$IQR = 15.5 - 13 = \boxed{2.5}$

CUTOFFS : $1.5 \times 2.5 = 3.75$

$Q_1 - 1.5 \times IQR = 13 - 3.75 = \boxed{9.25}$

$Q_3 + 1.5 \times IQR = 15.5 + 3.75 = \boxed{19.25}$

21 IS THE ONLY OUTLIER.



3. (4 points) A certain test has a mean of 79.728 and a standard deviation of 6.4. If your z-score on the test was 2.23, what was your actual test score?

$$\frac{X - 79.728}{6.4} = 2.23$$

$$\Downarrow$$

$$X - 79.728 = 14.272$$

$$\Rightarrow \boxed{X = 94}$$

4. (6 points) Refer to the chocolate chip counts given in Problem 2.

   (a) Find the percentile for a cookie with 14 chips.

$$\frac{\#\text{ OF DATA VALUES} < 14}{\#\text{ OF DATA VALUES}} \times 100\% = \frac{8}{24} \times 100\%$$

$$= 33.33\%$$

$$\approx 33^{\text{RD}} \text{ PERCENTILE}$$

   (b) Find the number of chips at the 65th percentile.

$$\frac{L}{24} = 0.65 \Rightarrow L = 15.6$$

ROUND UP TO $L = 16$

16^{TH} DATA VALUE IS **15**

5. (4 points) The odds against winning a certain game are 17 to 3. What is the probability of winning?

ODDS IN FAVOR ARE $\frac{3}{17}$

$\Rightarrow$ PROB IS $\frac{3}{20}$

6. (4 points) A PSC student is selected at random. Let A be the event that the student is taking a statistics class. Let B be the event that the student is a female. Are A and B disjoint? Explain.

NO, A PSC STUDENT CAN BE BOTH A STATISTICS STUDENT AND A FEMALE.

7. (10 points) Suppose A and B are events such that $P(A) = 0.85$, $P(B) = 0.55$, and $P(A \cup B) = 0.93$.

(a) Determine $P(A \cap B)$.

$$0.93 = 0.85 + 0.55 - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \boxed{0.47}$$

(b) Determine $P(\overline{B})$.

$$1 - 0.55 = \boxed{0.45}$$

(c) Find the odds in favor of A.

$$\frac{P(A)}{P(\overline{A})} = \frac{0.85}{0.15} = \frac{85}{15} = \boxed{\frac{17}{3}}$$

(d) Determine $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.47}{0.85} \approx 0.5529$$

(e) Are A and B independent? Explain.

NOT REALLY. $P(B|A) = 0.5529$

$\neq P(B) = 0.55$

CLOSE, BUT NOT EQUAL.

8. (4 points) Two people are selected at random. What is the probability that they were born on the same day of the week?

1ST PERSON HAS A BIRTHDAY ON SOME DAY OF THE WEEK

2ND PERSON HAS SAME DAY WITH PROB = $\boxed{\frac{1}{7}}$

9. (4 points) A card is selected at random from a standard deck. Let A be the event of drawing a red card. Let B be the event of drawing a jack. Are A and B independent? Explain.

$P(A) = \frac{1}{2}$, $P(B) = \frac{4}{52}$, $P(A \cap B) = \frac{2}{52}$

$P(A) \times P(B) = \frac{1}{2} \times \frac{4}{52} = \frac{2}{52} = P(A \cap B)$

YES, THEY ARE INDEPENDENT.

10. (3 points) When Sabrina asked to use the family car, her mother said, "No way! With all this snow and ice, there is a 95% chance you'll have an accident." What type of probability (theoretical, experimental, subjective, or geometric) did Sabrina's mother compute? Explain.

SUBJECTIVE --- SABRINA'S MOTHER DID NOT COUNT OR EXPERIMENT. SHE MADE A JUDGEMENT BASED ON HER KNOWLEDGE.

11. (5 points) If you make random guesses for 10 multiple-choice test questions (each with five possible answers), what is the probability of getting at least one correct?

PROB OF ALL WRONG $= \left(\frac{4}{5}\right)^{10} \approx 0.10737$

PROB OF AT LEAST ONE RIGHT $= 1 - \left(\frac{4}{5}\right)^{10}$

≈ 0.8926

12. (5 points) The five-number summary for a data set is:

Min $= 7$, $Q_1 = 26$, Med $= 43$, $Q_3 = 51$, Max $= 92$

What are the cutoff values for outliers? Does the data set have any outliers?

IQR $= 51 - 26 = 25$

$1.5 \times IQR = 37.5$

$Q_1 - 37.5 =$ $\boxed{-11.5}$

$Q_3 + 37.5 =$ $\boxed{88.5}$

YES! THERE IS AT LEAST ONE OUTLIER, NAMELY 92.

13. (10 points) After election day, a number of registered voters were surveyed. The data are given in the table below.

	Democrat	Republican
Voted	47	65
Did not vote	74	26

$47+65+74+26 = 212$

One of these registered voters is selected at random.

(a) What is the probability that the person is a Republican?

$$\frac{65+26}{212} = \boxed{\frac{91}{212}}$$

(b) What is the probability that the person did not vote?

$$\frac{74+26}{212} = \boxed{\frac{100}{212}}$$

(c) What is the probability that the person is a Republican who did not vote?

$$\boxed{\frac{26}{212}}$$

(d) What is the probability that the person did not vote given that the person is a Republican?

$$\frac{26}{65+26} = \boxed{\frac{26}{91}}$$

(e) Are $\underbrace{\text{voting}}_{A}$ and $\underbrace{\text{being Republican}}_{B}$ independent? Show work or explain.

$P(A) = \frac{47+65}{212} = \frac{112}{212}$

$P(B) = \frac{91}{212}$

$P(A \cap B) = \frac{65}{212} = 0.3066037\ldots$

$P(A) \times P(B) = \frac{112}{212} \times \frac{91}{212} = 0.2267710\ldots$

No, NOT INDEPENDENT.

14. (10 points) A letter is selected at random from the word *MISSISSIPPI.*

(a) What is the sample space for this experiment?

$\{M, I, S, P\}$.

(b) What is the probability of each outcome in your sample space?

$P(\{M\}) = \frac{1}{11}$ $\quad P(\{S\}) = \frac{4}{11}$

$P(\{I\}) = \frac{4}{11}$ $\quad P(\{P\}) = \frac{2}{11}$

(c) Are your probabilities above theoretical, experimental, or subjective? Explain.

THEORETICAL -- I ASSUMED EACH LETTER OF THE WORD IS EQUALLY LIKELY AND I COUNTED LETTERS.

(d) If B is the event of drawing the letter S, then what is the event $\overline{B}$?

$B = \{S\} \Rightarrow \overline{B} = \{M, I, P\}$

PROB IS $1 - \frac{4}{11} = \frac{7}{11}$

(e) Steve claims that the odds of selecting an S are 4/11. Is Steve correct? If not, correct him.

$P(\{S\}) = \frac{4}{11}$ PROB OF SELECTING S IS $\frac{4}{11}$, NOT ODDS.

ODDS ARE $\boxed{\frac{4}{7}}$

15. (9 points) Three letters are selected, without replacement, from the English alphabet. Let x represent the number of vowels that are selected. The probability distribution for x is shown below.

x	$P(x)$	
0	$\frac{7980}{15600}$	≈ 0.5115
1	$\frac{6300}{15600}$	≈ 0.4038
2	$\frac{1260}{15600}$	≈ 0.0808
3	$\frac{60}{15600}$	≈ 0.003846

(a) How can you be certain that the table describes a probability distribution?

EACH PROBABILITY IN THE DISTRIBUTION IS BETWEEN 0 & 1, AND THEY ADD UP TO 1:

$$\frac{7980}{15600} + \frac{6300}{15600} + \frac{1260}{15600} + \frac{60}{15600} = \frac{15600}{15600}$$

(b) What is the expected value of x?

$$\mu = 0\left(\frac{7980}{15600}\right) + 1\left(\frac{6300}{15600}\right) + 2\left(\frac{1260}{15600}\right) + 3\left(\frac{60}{15600}\right)$$

$$= \frac{15}{26} \approx 0.5769$$

(c) Are there any unusually large or small values of x?

USING THE CUMULATIVE 5% RULE,

$X = 3$ IS UNUSUALLY LARGE.

$\left(P(X \geq 3) \approx 0.4\%\right)$