Math 153 - Test 3a

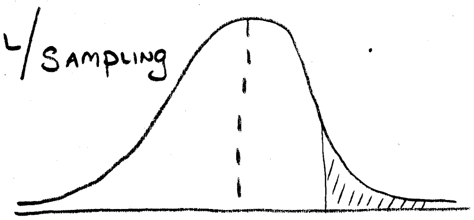
April 24, 2014

Name key Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) U.S. mortgage payments are approximately normally distributed with mean \$982 and standard deviation \$180. For a sample of 25 mortgage-paying families, what is the probability that their mean payment is greater than \$1100?

NORMAL/SAMPLING



$\mu_{\bar{x}} = \$982$

$\sigma_{\bar{x}} = \dfrac{\$180}{\sqrt{25}} = \$36$

normalcdf(1100, 99999, 982, 36)

$\approx \boxed{5.232 \times 10^{-4}}$

2. (4 points) A poison control center receives an average of 120 calls per month. What is the probability that the call center will receive more than 5 calls on any given day?

POISSON WITH

$\mu = \dfrac{120}{30} = 4$

$P(x > 5) = 1 - P(x \leq 5)$

$= 1 -$ poissoncdf(4, 5)

≈ 0.2149

$= \boxed{21.49\%}$

3. (3 points) Fifty-three percent of all people in the U.S. have at least some college education. In a random sample of 20 Americans, what would be an unusually large number of college-educated people?

BINOMIAL WITH

$p = 0.53$

$q = 0.47$

$N = 20$

$\mu = Np = 20(0.53) = 10.6$

$\sigma = \sqrt{Npq} = \sqrt{20(0.53)(0.47)} \approx 2.2$

$\mu + 2\sigma \approx 15$

15 OR MORE WOULD BE UNUSUALLY LARGE

4. (4 points) A sample of 500 nursing applications included 60 from men. Find a 90% confidence interval estimate for the true proportion of men who apply to nursing programs. Write a sentence that gives an interpretation of your result.

$X = 60$

$\hat{p} = 0.12$

1-PropZInt WITH

$X = 60$

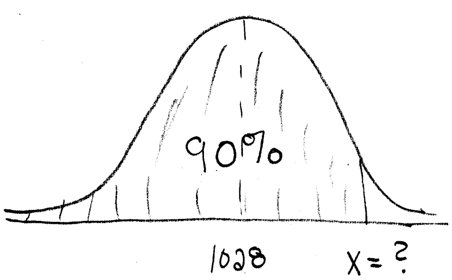
$N = 500$

C-LEVEL = 0.90

INTERVAL IS $(0.0961, 0.1439)$

WE CAN BE 90% CONFIDENT THAT THE TRUE PROPORTION OF MEN WHO APPLY TO NURSING PROGRAMS IS BETWEEN 0.0961 & 0.1439.

5. (3 points) The national average SAT score is 1028. If we assume that scores are normally distributed with $\sigma = 92$, then what score is at the 90th percentile?



invNorm(0.90, 1028, 92)

$\approx \boxed{1145.9}$

6. (3 points) A political pollster wants to construct a 95% confidence interval estimate for the true proportion of Americans who approve of the Affordable Care Act. If she wants a margin of error of 3%, how many people should be surveyed?

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025 \Rightarrow z_{\alpha/2} = \text{invNorm}(0.975)$

$\approx$

$$\text{SAMPLE SIZE} = N = \frac{(1.96)^2(0.25)}{(0.03)^2} \approx 1067.11$$

$\boxed{1068 \text{ PEOPLE}}$

7. (2 points) When sampling, not all sampling statistics (estimators) target their corresponding population parameters. List two estimators that target their corresponding population parameters and one that does not.

TARGET	DOES NOT TARGET
MEAN	RANGE
PROPORTION	

8. (2 points) Seventy percent of all single-vehicle traffic fatalities that occur on weekend nights involve intoxicated drivers. In a sample of 15 such traffic fatalities, what is the probability that exactly 12 involved an intoxicated driver?

BINOMIAL WITH

$p = 0.70$

$N = 15$

$P(x = 12) = \text{binomialpdf}(15, 0.70, 12)$

≈ 0.1700

$= \boxed{17\%}$

Math 153 - Test 3b

April 24, 2014

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Tuesday, April 29. You must work individually on this test—you will not be given any credit for group work.

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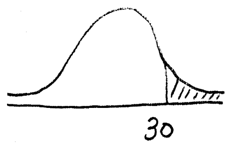
1. (12 points) In 2005, the average commute to work was 25 minutes with a standard deviation of 6.1 minutes. Assume that commute times are normally distributed.

   (a) What is the probability that a randomly selected commute time exceeds 30 minutes?

   $\mu = 25$

   $\sigma = 6.1$

   $P(x > 30)$

   $= \text{normalcdf}(30, 99999, 25, 6.1)$

   $\approx \boxed{0.2062}$

   (b) In a random sample of 500 commute times, about how many of them will exceed 30 minutes?

   $500(0.2062) \approx \boxed{103}$

   (c) A random sample of 16 commute times is obtained. What is the probability that the mean of the 16 commute times exceeds 30 minutes?

   $\mu_{\bar{x}} = 25$

   $\sigma_{\bar{x}} = \dfrac{6.1}{\sqrt{16}} = 1.525$

   $P(\bar{x} > 30)$

   $= \text{normalcdf}(30, 99999, 25, 1.525)$

   $\approx \boxed{5.215 \times 10^{-4}}$

   (d) In a random sample of 16 commute times, what would be an unusually large mean commute time?

   $\mu_{\bar{x}} + 2\sigma_{\bar{x}} = 25 + 2(1.525) = 28.05$ min

   Anything bigger than $\boxed{28.05 \text{ min}}$ would be an unusually large mean.

2. (7 points) A survey of 30 adults found that the mean age of a person's primary vehicle is 5.6 years. Assuming the standard deviation of the population is 0.8 year, find a 99% confidence interval estimate for the true population mean. Write a sentence that gives an interpretation of your result.

$N = 30$

$\bar{X} = 5.6$

$\sigma = 0.8$

SAMPLING DIST IS ROUGHLY NORMAL SINCE $N = 30$

ZInterval

Stats

$\sigma = 0.8$

$\bar{X} = 5.6$

$n = 30$

C-Level = 0.99

99% C.I. IS

$(5.2238, 5.9762)$

WE ARE 99% CONFIDENT THAT THE TRUE POPULATION MEAN LIES IN THIS INTERVAL.

3. (4 points) A scientist wishes to determine the average depth of a river. He wants to be 99% confident that his estimate is accurate to within 2 feet. From a previous study, the standard deviation of the depths was found to be 4.38 feet. How many sample measurements must the scientist obtain?

$$99\% \text{ C.I.} \Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005$$

$$Z_{\alpha/2} = \text{invNorm}(0.995) \approx 2.576$$

$$N = \left(\frac{2.576 \cdot 4.38}{2}\right)^2 = 31.825\ldots \approx 32$$

$\boxed{\text{32 SAMPLES}}$

4. (4 points) After surveying 1002 people, a 98% confidence interval estimate for the true proportion of people who voted in the last election was found to be (66.6%, 73.3%). Fred claims that there is a 98% chance that the true proportion is in that interval. Is Fred's claim correct? Explain your reasoning.

FRED'S CLAIM IS INCORRECT. THE TRUE PROPORTION IS IN THERE OR NOT --- NO PROBABILITY ASSOCIATED WITH THAT.

IT WOULD BE TRUE FOR FRED TO SAY HE IS 98% CONFIDENT THE TRUE PROPORTION IS IN THE INTERVAL.

5. (12 points) A jar contains one penny, one nickel, and one dime. Recall that the *midrange* of a set of numbers is one-half of the sum of the minimum and maximum values.

(a) Find the midrange of the values of coins in the jar.

$$\frac{1+10}{2} = \frac{11}{2} = \boxed{5.5\text{¢}}$$

(b) Two coins are selected from the jar (without replacement). List all possible two-coin samples as ordered pairs of the form (*first coin*, *second coin*). Find the midrange of each sample.

(1,5) --- 3¢ (5,1) --- 3¢ (10,1) --- 5.5¢

(1,10) --- 5.5¢ (5,10) --- 7.5¢ (10,5) --- 7.5¢

(c) Summarize the sampling distribution of midranges in a probability distribution table.

X	P(x)
3	$\frac{2}{6}$
5.5	$\frac{2}{6}$
7.5	$\frac{2}{6}$

(d) Find the mean (i.e. $\mu = \sum x \cdot P(x)$) of the sample midranges.

$$\mu = 3\left(\frac{2}{6}\right) + 5.5\left(\frac{2}{6}\right) + 7.5\left(\frac{2}{6}\right) = \boxed{5.\overline{3}\text{¢}}$$

(e) Do the sample midranges target the population midrange? Explain.

No, SAMPLE MIDRANGES TARGET $5.\overline{3}$¢, NOT THE POPULATION MIDRANGE 5.5¢.

6. (12 points) A certain vehicle emissions inspection facility inspects roughly 67 cars per day.

   (a) An additional employee is required if the number of daily inspections exceeds 75. On any given day, what is the probability that an additional employee will be required?

Poisson

$\mu = 67$

$P(x > 75) = 1 - P(x \le 75)$

$= 1 - \text{poissoncdf}(67, 75)$

$\approx \boxed{0.1497}$

   (b) What is an usually large number of daily inspections?

$\mu = 67$

$\sigma = \sqrt{67}$

$\mu + 2\sigma \approx \boxed{83.37}$

84 INSPECTIONS IS UNUSUALLY LARGE

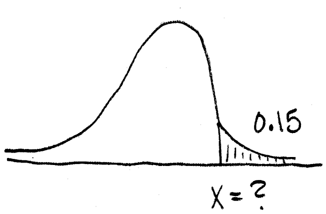
   (c) Your calculator does not have a feature for solving inverse Poisson problems. Nonetheless, you can use `poissoncdf` with trial and error to solve inverse problems. Do this to approximate the number of daily inspections that is at the 93rd percentile.

$\text{poissoncdf}(67, 78) \approx 0.9173$

$\text{poissoncdf}(67, 79) \approx 0.9336$

$\boxed{\text{79 INSPECTIONS}}$ AT 93RD PERCENTILE

7. (5 points) In a certain community, new home prices are normally distributed with mean \$224,000 and standard deviation \$14,000. A developer wishes to build and sell "affordable" homes, so he pledges to only price his homes at a level below that of the most expensive 15% of homes. What is the highest priced home the developer will list?



$\text{invNorm}(0.85, 224000, 14000)$

$\approx \boxed{\$238{,}510.07}$

8. (6 points) The sodium content in a certain brand of microwave dinner is 660 mg with a standard deviation of 35 mg. In a random sample of 45 dinners, find the probability that the mean sodium content is less than 650 mg.

$\mu_{\bar{x}} = 660$

$\sigma_{\bar{x}} = \frac{35}{\sqrt{45}}$

$P(\bar{x} < 650) = \text{normalcdf}\left(-99999, 650, 660, \frac{35}{\sqrt{45}}\right)$

$\approx \boxed{0.0276}$

9. (7 points) It has been reported that about 11% of U.S. elementary students attend private school. A random sample of 450 students indicated that 55 attended private school. Use a 95% confidence interval to estimate the true population proportion of students attending private school. What can you say about the accuracy of the reported percentage?

1-PropZInt

$X = 55$

$n = 450$

C-Level $= 0.95$

WE CAN BE 95% CONFIDENT THAT THE TRUE POPULATION PROPORTION IS BETWEEN 0.09196 AND 0.15249.

SINCE 11% IS INSIDE OUR C.I., IT IS PROBABLY AN ACCURATE ESTIMATE.

10. (6 points) Approximately 10.3% of American high school students drop out of school before graduation. In a random sample of 18 students entering high school, what is the probability more than 3 will eventually drop out?

BINOMIAL

$p = 0.103$

$q = 0.897$

$N = 18$

$P(x > 3) = 1 - P(x \leq 3)$

$= 1 - \text{binomialcdf}(18, 0.103, 3)$

$\approx \boxed{0.1068}$