Math 153 - Final Exam

May 15, 2014

Name key

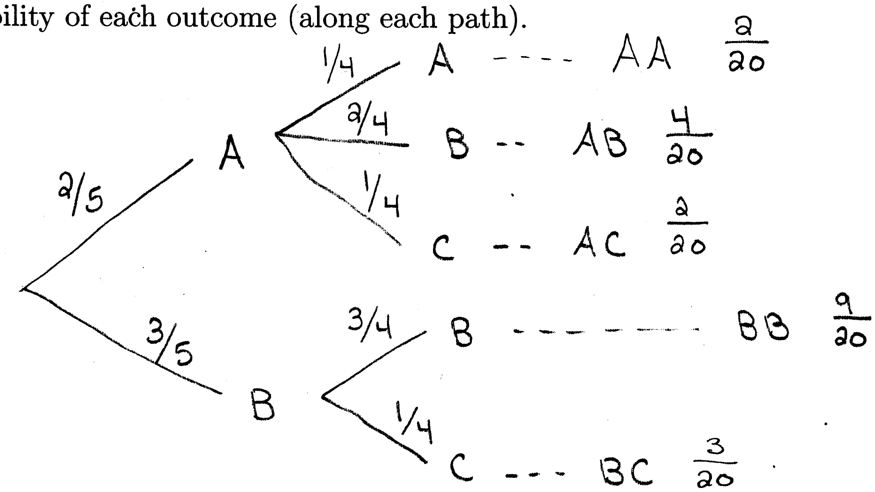
Score

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected from the second box.

A A B B B		B B C

(a) Sketch the probability tree associated with this two-stage experiment and find the probability of each outcome (along each path).



(b) Are the probabilities above theoretical or experimental? Explain your reasoning.

THEORETICAL, WE DID NOT DO THE EXPERIMENT. WE ASSUMED EACH OBJECT IS EQUALLY LIKELY, AND WE COUNTED OBJECTS.

(c) What is the probability of selecting the letter B from the second box?

AB, BB

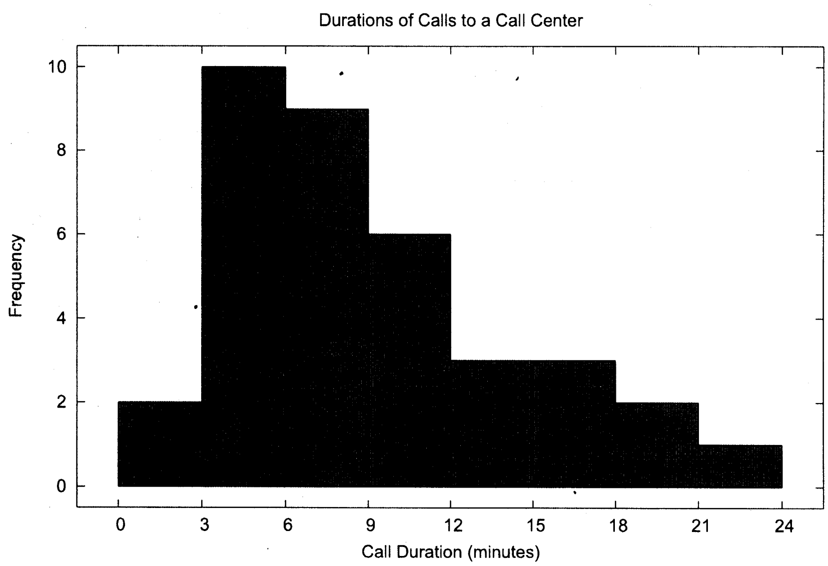
$$\frac{4}{20} + \frac{9}{20} = \boxed{\frac{13}{20}}$$

(d) What are the odds against selecting a B from the second box?

PROB IS $\frac{13}{20}$ $\Rightarrow$ ODDS IN FAVOR ARE $\frac{13}{7}$.

$\Rightarrow$ ODDS AGAINST ARE $\boxed{\frac{7}{13}}$

2. (6 points) The histogram below shows the distribution of the lengths of phone calls (in minutes) to a customer service center on a particular day.



(a) How many phone calls did the service center receive on that particular day?

$$2+10+9+6+3+3+2+1 = \boxed{36}$$

(b) If the histogram was changed to a relative frequency histogram, what would be the height of the third bar?

$$\frac{9}{36} = \frac{1}{4} = \boxed{0.25}$$

(c) Are the lengths of calls normally distributed? Explain. If not, describe the distribution.

NO. THEY DO NOT APPEAR NORMAL. THE DISTRIBUTION IS SKEWED RIGHT.

3. (5 points) Refer to Problem 2. Use class midpoints to compute the mean duration of the service center's calls.

$$\frac{1.5(2)+4.5(10)+7.5(9)+10.5(6)+13.5(3)+16.5(3)+19.5(2)+22.5(1)}{36}$$

$$= \frac{330}{36} \approx \boxed{9.17}$$

4. (12 points) ACT scores have mean 18 and standard deviation 6. SAT scores have mean 500 and standard deviation 110

   (a) Compute the coefficients of variation for the different tests. Is there more spread in ACT scores or SAT scores?

   ACT: $\frac{6}{18} \approx \boxed{0.3}$ SAT: $\frac{110}{500} = \boxed{0.22}$ MORE SPREAD IN ACT.

   (b) Compute z-scores to determine which is "relatively" greater, an ACT score of 28 or an SAT score of 698.

   ACT: $z = \frac{28-18}{6} \approx \boxed{1.67}$

   SAT: $z = \frac{698-500}{110} = \boxed{1.8}$

   THE SAT SCORE IS GREATER.

   (c) What is an unusually high ACT score?

   $18 + 2(6) = \boxed{30}$

5. (10 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

   (a) Compute $P(A)$. $1 - 0.52 = \boxed{0.48}$

   (b) Compute $P(A \cap B)$. $= P(A) + P(B) - P(A \cup B)$

   $= 0.48 + 0.55 - 0.766 = \boxed{0.264}$

   (c) Compute $P(B|A)$.

   $= \frac{P(A \cap B)}{P(A)} = \frac{0.264}{0.48} = \boxed{0.55}$

   (d) Are A and B independent? Explain.

   $\boxed{\text{YES,}}$ SINCE $P(B|A) = P(B)$.

   (e) What are the odds in favor of A?

   $\frac{0.48}{0.52} = \frac{48}{52} = \boxed{\frac{12}{13}}$

6. (16 points) According to the Bureau of Labor Statistics, the average hourly wages (in dollars) for nine law-related occupations are:

75, 38, 27, 48, 23, 23, 20, 20, 26

(a) Find the range and the sample standard deviation.

RANGE $= 75 - 20$
$= \boxed{55}$

FROM CALCULATOR ...
$\boxed{S \approx 18.15}$

(b) Find the median, quartiles, and the interquartile range.

CALCULATOR ...

$\boxed{\bar{X} = 33.\bar{3}}$, $\boxed{Q_1 = 21.5}$, $\boxed{Q_2 = 26}$, $\boxed{Q_3 = 43}$, $\boxed{IQR = Q_3 - Q_1 = 21.5}$

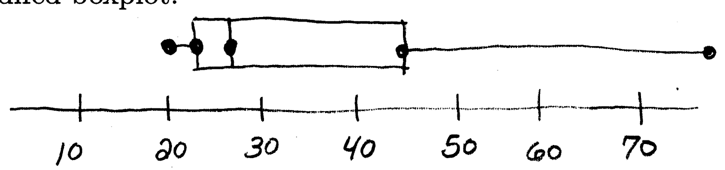
(c) Compute the cut-off values for outliers.

$Q_1 - 1.5 \times IQR = 21.5 - 1.5(21.5) = \boxed{-10.75}$

$Q_3 + 1.5 \times IQR = 43 + 1.5(21.5) = \boxed{75.25}$

NO OUTLIERS

(d) Sketch the modifed boxplot.



7. (12 points) The monthly amounts of paper waste generated by American households are normally distributed with mean 28 lbs and standard deviation 2 lbs.

(a) What amount of paper waste is at the 85th percentile?

invNorm(0.85, 28, 2) $\approx$ $\boxed{30.07 \text{ lbs}}$

(b) What is an unusually small amount of paper waste?

$28 - 2(2) = \boxed{24 \text{ lbs}}$

(c) In a sample of 700 households, about how many would generate more than 31.2 lbs of paper waste?

700 × normalcdf(31.2, 99999, 28, 2)

$\approx$ $\boxed{38 \text{ HOUSEHOLDS}}$

8. (16 points) In a probability experiment, the random variable x has five possible values: 1, 2, 4, 5, 6. The (incomplete) probability distribution for x is shown below.

x	$P(x)$	
1	0.262	
2	0.471	
4	0.034	
5	???	$\boxed{0.211}$
6	0.022	

(a) Determine the missing probability, $P(5)$.

$$P(5) = 1 - (0.262 + 0.471 + 0.034 + 0.022)$$

$$= \boxed{0.211}$$

(b) Compute the mean value of the random variable.

$$\mu = 1(0.262) + 2(0.471) + 4(0.034) + 5(0.211) + 6(0.022)$$

$$= \boxed{2.527}$$

(c) Compute the standard deviation in the values of x.

$$\sigma^2 = 1(0.262) + 4(0.471) + 16(0.034) + 25(0.211) + 36(0.022) - (2.527)^2$$

$$= 2.371271$$

$$\boxed{\sigma \approx 1.54}$$

(d) Determine the unusually small and large values of x.

SINCE $P(x \geq 6) = 0.022 < 0.05$,

$\boxed{6}$ IS UNUSUALLY LARGE.

9. (12 points) According to the results of a 2008 survey, 60% of all adult U.S. residents are very satisfied with the lives they lead. A random sample of 1010 adult U.S. residents is selected.

   (a) Find the probability that at least 620 of the people in the sample are very satisfied with their lives.

   $$P(x \geq 620) = 1 - P(x < 619)$$

   $$= 1 - \text{binomialcdf}(1010, 0.60, 619) \approx \boxed{0.1931}$$

   (b) In that sample, what would be an unusually small number of very satisfied people?

   $$\mu = 1010 \cdot (0.60) = 606 \qquad \mu - 2(\sigma)$$

   $$\sigma = \sqrt{1010\,(0.60)(0.40)} \approx 15.5692 \qquad \approx 574.86 \approx \boxed{575}$$

   (c) Find the probability that exactly 600 of the people in the sample are very satisfied with their lives.

   $$P(x=600) = \text{binomialpdf}(1010, 0.60, 600) \approx \boxed{0.0237}$$

10. (12 points) The mean yearly Medicare spending per beneficiary is \$5694. Suppose the spendings are normally distributed with standard deviation \$612. A random sample of 15 patients is obtained.

   (a) What is the standard deviation of the sampling distribution?

   $$\sigma_{\bar{x}} = \frac{612}{\sqrt{15}} \approx \boxed{158.02}$$

   (b) What is the probability that the sample mean is less than \$4800?

   $$\text{normalcdf}\left(-99999, 4800, 5694, \frac{612}{\sqrt{15}}\right)$$

   $$\approx \boxed{7.697 \times 10^{-9}}$$

   (c) What would be an usually large sample mean?

   $$5694 + 2(158.02)$$

   $$= \boxed{6010.04}$$

11. (10 points) The bottlers of YumTea know that the volumes of their bottles are normally distributed, but they have no reliable estimate for the population standard deviation. The volumes (in ml) of ten randomly selected bottles are given below.

502.9, 499.8, 503.2, 502.8, 500.9, 503.9, 498.2, 502.5, 503.8, 501.4

(a) Construct a 95% confidence interval estimate for the mean volume of bottles of YumTea.

TInterval

Data

95% C-Level

$\bar{x} = 501.94$

$S_x \approx 1.85$

$n = 10$

95% C.I. is

(500.62, 503.26)

(b) Determine the sample size required to have a confidence interval estimate with a margin of error of ±1 ml at the level $\alpha = 0.05$. (Use the sample standard deviation to estimate the population standard deviation.)

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 \approx \left(\frac{1.96\,(1.85)}{1}\right)^2 \approx \boxed{14}$$

12. (15 points) The makers of YumTea claim that the majority of tea drinkers prefer YumTea over Nestea. In a random sample of 20 tea drinkers, 13 preferred YumTea. Test the claim made by the makers of YumTea at the level $\alpha = 0.05$.

(a) State the null and alternative hypotheses.

Claim: $p > 0.50$

Opposite Claim: $p \leq 0.50$

$H_0: p = 0.50$

$H_1: p > 0.50$

(b) Compute the test statistic.

$$\hat{p} = \frac{13}{20} = 0.65$$

$$z = \frac{0.65 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{20}}} \approx \boxed{1.34164}$$

(c) Find the P-value and draw a conclusion about YumTea's claim.

P-value $\approx 0.0898563 > 0.05$

We cannot reject H_0. There is not sufficient evidence to support the original claim.

13. (12 points) The data below were obtained in a study of research spending at American colleges and universities.

Year x	Spending (billions of dollars) y
2003	40.1
2004	43.3
2005	45.8
2006	47.7
2007	49.4

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between x and y.

LinReg $(ax+b)$

$\boxed{r \approx 0.9911}$

SINCE THIS IS VERY CLOSE TO 1,
THERE IS A VERY STRONG POSITIVE
LINEAR CORRELATION BETWEEN X & Y.

(b) Find the regression equation.

$y = 2.3x - 4566.24$

(c) Use your regression equation to predict the 2008 spending.

$2.3(2008) - 4566.24$

$= 52.16$

$\approx$ $\boxed{\$52.2 \text{ BILLION}}$