Math 153 - Test 3a

April 16, 2015

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (3 points) In the United States, a person currently in his/her early 20's has an 80% chance of being alive at age 65. In a group of 50 twenty-year-olds, what would be an unusually small number of those who survive to 65?

BINOMIAL
$n = 50$
$p = 0.80$
$q = 0.20$

$$\mu - 2\sigma = 50(0.80) - 2\sqrt{50(0.80)(0.20)}$$
$$\approx 34.34$$

$\boxed{34 \text{ OR FEWER}}$

2. (3 points) In the state of Illinois, there are, on average, 30 days each year that the daily high temperature exceeds 90°. In any given year, what is the probability that this temperature will be exceeded on at least 35 days?

POISSON
$\mu = 30$

$$P(x \geq 35) = 1 - P(x < 35) = 1 - P(x \leq 34)$$
$$= 1 - \text{poissoncdf}(30, 34)$$
$$\approx \boxed{0.2027}$$

3. (3 points) Given the following discrete probability distribution, determine the unusually small and large values of x.

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.08	0.02	0.05	0.27	0.04	0.48	0.03	0.01	0.02

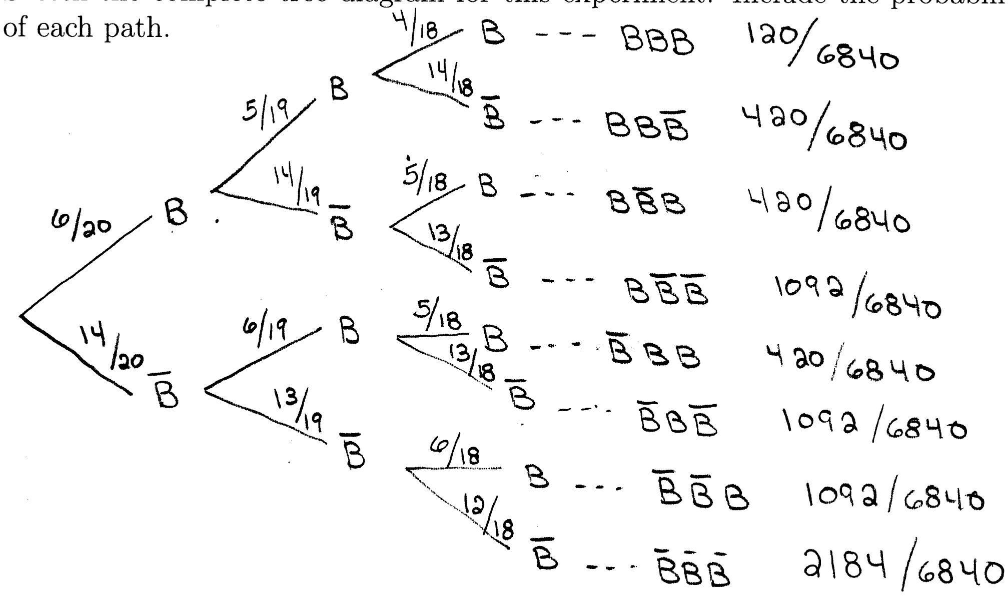
NO UNUSUALLY SMALL VALUES.

7 & 8 ARE UNUSUALLY LARGE BECAUSE

$P(x \geq 7) = 0.03 < 0.05$.

4. (10 points) A jar contains 20 marbles, 6 of which are blue. Three marbles are selected at random **without replacement**. Let the random variable x represent the number of blue marbles in the sample of 3.

(a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



(b) The random variable x (representing the number of blue marbles) can take four different values: 0, 1, 2, 3. Use your tree to determine the probability distribution for x.

X	0	1	2	3
P(x)	$\frac{2184}{6840}$	$\frac{3276}{6840}$	$\frac{1260}{6840}$	$\frac{120}{6840}$

(c) Find the mean value of x.

$$\mu = \frac{0(2184) + 1(3276) + 2(1260) + 3(120)}{6840} = \frac{6156}{6840} = \boxed{0.9}$$

(d) Find the standard deviation in x.

$$\sigma^2 = \frac{0(2184) + 1(3276) + 4(1260) + 9(120)}{6840} - (0.9)^2$$

$$\approx 0.56368$$

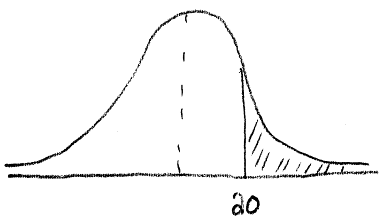
$$\Rightarrow \boxed{\sigma \approx 0.75}$$

(e) What are the unusually large values of x?

$$0.9 + 2(0.75) = 2.4$$

$\boxed{\text{3 BLUES ARE UNUSUAL}}$

5. (3 points) Carapace lengths of adult male Brazilian tawny red tarantulas are normally distributed with mean 18.14 mm and standard deviation 1.76 mm. In a sample of 250 of these tarantulas, about how many will have a carapace longer than 20 mm?



$250 \cdot \text{normalcdf}(20, 99999, 18.14, 1.76)$

≈ 36.32

NORMAL

$\mu = 18.14$

$\sigma = 1.76$

$\boxed{\text{ABOUT } 36}$

6. (3 points) Approximately 10% of the world's population are left-handed. However, there is a general tendency that the more violent a particular society is, the higher the proportion of left-handers. For example, there is a very high homicide rate among the Eipo of Indonesia, and 27% of the Eipo are left-handed. In a group of 35 Eipo, what is the probability that fewer than 8 are left-handed?

BINOMIAL

$n = 35$

$p = 0.27$

$P(X<8) = \text{binomcdf}(35, 0.27, 7)$

$\approx \boxed{0.2333}$

Math 153 - Test 3b

April 16, 2015

Name key Score

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Tuesday, April 21. YOU MUST WORK INDIVIDUALLY ON THIS TEST—YOU WILL NOT BE GIVEN ANY CREDIT FOR GROUP WORK.

1. (9 points) In the United States, 1 in 6 people have light blue eyes. 75 Americans are randomly selected.

BINOMIAL
$n = 75$
$p = 1/6$
$q = 5/6$

(a) What is the probability that 15 have light blue eyes?

$$P(x = 15) = \text{binompdf}(75, 1/6, 15)$$

$$\approx \boxed{0.0861}$$

(b) What is the probability that more than 20 have light blue eyes?

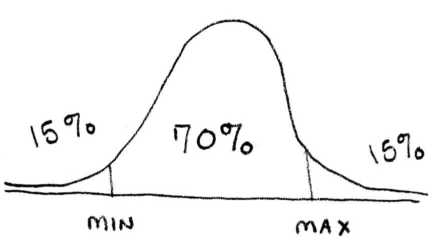
$$P(x > 20) = 1 - P(x \leq 20) = 1 - \text{binomcdf}(75, 1/6, 20)$$

$$\approx \boxed{0.0095}$$

(c) How many people in the sample are expected to have light blue eyes?

$$\mu = np = 75\left(\tfrac{1}{6}\right) = \boxed{12.5}$$

2. (6 points) An automobile dealer finds that used car prices are normally distributed with mean \$15,700 and standard deviation \$2150. The dealer decides to sell cars that appeal to the middle 70% of the market in terms of price. Find the minimum and maximum prices of the cars the dealer will sell.



MIN PRICE

$$= \text{invNorm}(0.15, 15700, 2150)$$

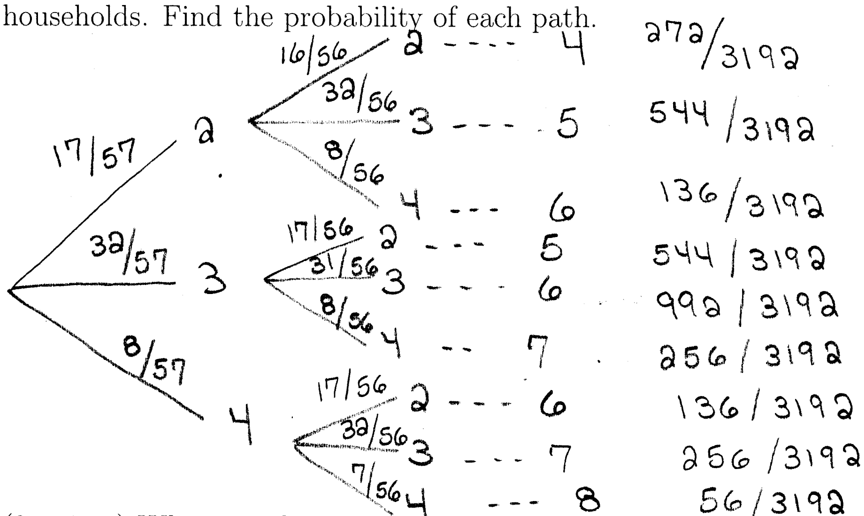
$$\approx \boxed{13,471.67}$$

MAX PRICE $= \text{invNorm}(0.85, 15700, 2150)$

$$\approx \boxed{17,928.33}$$

3. In a certain neighborhood, there are 17 two-person households, 32 three-person households, and 8 four-person households. Two households are picked at random without replacement. Let the random variable x represent the total number of people in the two households.

   (a) (5 points) Sketch the probability tree associated with the selection of the two households. Find the probability of each path.



   (b) (2 points) What are the possible values of the random variable x?

$4, 5, 6, 7, 8$

   (c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	4	5	6	7	8
P(x)	$\frac{272}{3192}$	$\frac{1088}{3192}$	$\frac{1264}{3192}$	$\frac{512}{3192}$	$\frac{56}{3192}$

   (d) (3 points) Find the mean value of x.

$$\mu = 4\left(\frac{272}{3192}\right) + 5\left(\frac{1088}{3192}\right) + \dots + 8\left(\frac{56}{3192}\right) = \boxed{\frac{18144}{3192} \approx 5.68}$$

   (e) (3 points) Find the standard deviation in the values of x.

$$\sigma^2 = 16\left(\frac{272}{3192}\right) + 25\left(\frac{1088}{3192}\right) + \dots + 64\left(\frac{56}{3192}\right) - \left(\frac{18144}{3192}\right)^2 \approx 0.81$$

$$\boxed{\sigma \approx 0.90}$$

4. (9 points) In the United States, there are an average of 34 people per square kilometer. Suppose a random square kilometer of the United States is selected.

Poisson $\mu = 34$

   (a) What is the probability that more than 44 people live in that region?

$$P(x > 44) = 1 - P(x \le 44)$$

$$= 1 - \text{poissoncdf}(34, 44) \approx \boxed{0.0404}$$

   (b) What would be an unusually small population for that region?

$$\mu - 2\sigma = \mu - 2\sqrt{\mu} = 34 - 2\sqrt{34} \approx 22.34$$

22 or fewer

   (c) Your calculator does not have a feature for solving inverse Poisson problems. Nonetheless, you can use `poissoncdf` with trial and error to solve inverse problems. Do this to approximate the population of a square kilometer that is at the 80th percentile.

$$\text{poissoncdf}(34, x) \approx 0.80$$

$\boxed{X = 38}$ makes this closest to 0.80

5. (9 points) As reported in *Runners World* magazine, the times of the finishers in the New York City 10-km run are normally distributed with mean 61 minutes and standard deviation 9 minutes.

   (a) What is the probability that a randomly selected runner finished the race is less than 75 minutes?

$$\text{normalcdf}(-99999, 75, 61, 9) \approx \boxed{0.9401}$$

   (b) Neil Malcolm finished the race in 32.5 min. Is his time unusual?

$$\text{normalcdf}(-99999, 32.5, 61, 9) \approx 7.7 \times 10^{-4}$$

much less than 0.05

Very unusual!

   (c) What finish time is at the 75th percentile?

$$\text{invNorm}(0.75, 61, 9)$$

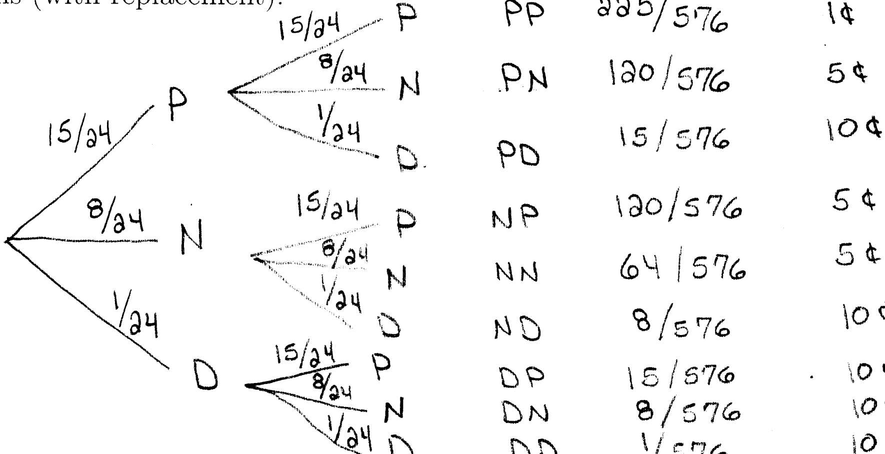
$$\approx \boxed{67.07 \text{ min}}$$

6. A jar contains 15 pennies, 8 nickels, and 1 dime. A person who does not know the contents of the jar would like to determine the highest-valued coin by sampling. Samples of two coins are selected **with replacement**.

(a) (1 point) What is the highest-valued coin in the population? (This is the population maximum.)

10¢ (Dime)

(b) (5 points) Sketch the complete tree diagram associated with selecting a sample of two coins (with replacement).



(c) (4 points) Let x be the maximum value of the two-coin sample. Use your tree diagram to determine the sampling distribution for x.

X	1	5	10
P(x)	$\frac{225}{576}$	$\frac{304}{576}$	$\frac{47}{576}$

(d) (3 points) What is the mean value of your sampling distribution?

$$\mu = \frac{1(225) + 5(304) + 10(47)}{576} = \frac{2215}{576} \approx 3.85$$

(e) (2 points) Do sample maximum values target the population maximum value? Explain your reasoning.

$3.85 \neq 10$

NO, THE MEAN OF THE SAMPLING DISTRIBUTION DOES NOT EQUAL THE POPULATION PARAMETER.

(f) (2 points) Is sampling a good way to determine a population max?

No! See (e)

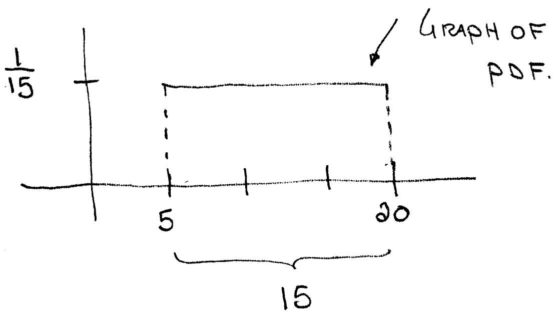
7. (8 points) A computer program generates random real numbers that are uniformly distributed between 5 and 20.

(a) If 1000 numbers were generated, would you expect more of them to be between 5 and 13 or between 13 and 20? Explain.

$13 - 5 = 8$ $\qquad$ $20 - 13 = 7$

MORE WOULD BE BETWEEN 5 AND 13 BECAUSE THE INTERVAL IS LONGER.

(b) About what percent of the generated numbers will be between 11 and 17?



$$\frac{1}{15}(17-11) = \frac{6}{15} = 0.4$$

$$\boxed{40\%}$$

(c) What generated number will be at the 90th percentile?

FIND X SO THAT

$$\frac{1}{15}(X-5) = 0.90$$

$$X - 5 = 13.5$$

$$\boxed{X = 18.5}$$