Math 153 - Final Exam

May 14, 2015

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all computations. Be sure to describe your calculator input when appropriate.

1. (10 points) The following frequency distribution shows the costs (in dollars) of 30 portable GPS navigators.

GPS Costs ($)	Frequency
65–104	6
105–144	9
145–184	6
185–224	4
225–264	2
265–304	1
305–344	2

(a) What are the class boundaries associated with the **first** class listed above?

$\boxed{64.5 \text{ AND } 104.5}$

(b) What is the class width?

$105 - 64 = \boxed{40}$

(c) What are the class midpoints?

$\frac{104+65}{2} = 84.5,\ 124.5,\ 164.5,\ 204.5,\ 244.5,\ 284.5,\ 324.5$

(d) Use class midpoints to estimate the (weighted) mean cost.

$$\frac{6(84.5) + 9(124.5) + \dots + 2(324.5)}{30} = \frac{4855}{30} \approx \$161.83$$

2. (16 points) The numbers of tornadoes in Illinois for each year from 1990 to 2000 are shown below.

$$50, 32, 23, 34, 20, 76, 62, 29, 99, 64, 55$$

(a) Find the range and the sample standard deviation.

Range $= 99 - 20 = \boxed{79}$

$S \approx 24.675.$

(b) Find the median, quartiles, and the interquartile range.

Med $= 50$

$Q_1 = 29$

$Q_3 = 64$

IQR $= 35$

(c) Compute the cut-off values for outliers.

$Q_1 - 1.5(IQR) = -23.5$

$Q_3 + 1.5(IQR) = 116.5$

(d) In any given year, what is an unusually small number of tornadoes?

$\bar{X} \approx 49.455$

$\bar{X} - 2s \approx 0.105$

3. (6 points) For Yellowstone's Old Faithful geyser, the mean time between eruptions is 1.55 hr with a standard deviation of 0.11 hr. For Yellowstone's Lone Star geyser, the mean is 3.00 hr with a standard deviation of 0.16 hr. Compute the coefficient of variation (CV) for each geyser. Which geyser's eruption cycle has more variation?

O.F. CV $= \frac{0.11}{1.55} \approx 7.1\%$

L.S. CV $= \frac{0.16}{3.00} \approx 5.3\%$

Old Faithful has more variation.

4. (15 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

(a) Compute $P(A)$.

$1 - 0.52 = \boxed{0.48}$

(b) Compute $P(A \cap B)$.

$= P(A) + P(B) - P(A \cup B) = 0.48 + 0.55 - 0.766$

$= \boxed{0.264}$

(c) Compute $P(B|A)$.

$= \dfrac{P(A \cap B)}{P(A)} = \boxed{0.55}$

(d) Are A and B independent? Explain.

YES! $P(B|A) = 0.55 = P(B)$

(e) What are the odds in favor of A?

$\dfrac{0.48}{0.52} = \dfrac{48}{52} = \boxed{\dfrac{12}{13}}$

5. (4 points) In January 2014, the mean maximum daily temperature was 21.8°F with a standard deviation of 12.8°F. Compute the z-score for −6.9°F. Do you think that −6.9°F was an unusually low temperature? Explain.

$z = \dfrac{-6.9 - 21.8}{12.8} \approx \boxed{-2.24}$

−6.9°F IS UNUSUALLY BECAUSE ITS Z-SCORE IS LESS THAN −2.

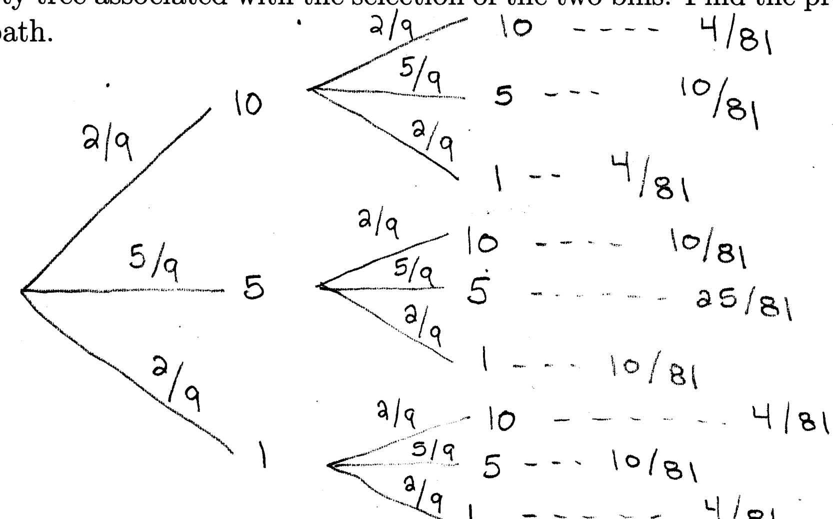
(IT IS MORE THAN 2 STD. DEVS BELOW THE MEAN.)

6. A box contains 2 ten-dollar bills, 5 five-dollar bills, and 2 one-dollar bills.

(a) (3 points) What is the mean value of the bills in the box?

$$\mu = \frac{2(10) + 5(5) + 2(1)}{9} = \frac{47}{9} = \$5.\overline{2}$$

(b) (6 points) Two bills are selected at random **with replacement**. Sketch the probability tree associated with the selection of the two bills. Find the probability of each path.



(c) (4 points) For each possible two-bill selection, let the random variable x represent the mean value of the two-bill sample. What are the possible values of x?

$X = 1, 3, 5, 5.5, 7.5, 10$

(d) (5 points) Use the probabilities from your tree to determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	1	3	5	5.5	7.5	10
P(x)	$\frac{4}{81}$	$\frac{20}{81}$	$\frac{25}{81}$	$\frac{8}{81}$	$\frac{20}{81}$	$\frac{4}{81}$

(e) (4 points) Find the mean value of the random variable x.

$$\mu_{\bar{x}} = \frac{4}{81} + 3\left(\frac{20}{81}\right) + 5\left(\frac{25}{81}\right) + 5.5\left(\frac{8}{81}\right) + 7.5\left(\frac{20}{81}\right) + 10\left(\frac{4}{81}\right)$$

$$\frac{423}{81} = \$5.\overline{2}$$

(f) (3 points) Compare your results from parts (a) and (e). Do sample means target the population mean? Did you expect this? Briefly explain.

$\mu = \mu_{\bar{x}}$

YES, THE MEAN OF THE SAMPLING DISTRIBUTION IS EQUAL TO THE POP. MEAN. YES, WE EXPECT THIS! (CENTRAL LIMIT THM)

7. (10 points) According to the *Humane Society of the United States*, 40% of all U.S. households own at least one dog. A sample of 20 U.S. households is selected at random.

   (a) What is the probability that exactly 9 of the households have at least one dog?

binomial

$n = 20$

$p = 0.40$

$q = 0.60$

$$P(x=9) = \text{binompdf}(20, 0.40, 9) \approx \boxed{0.1597}$$

   (b) What is the probability that at least 10 households have at least one dog?

$$P(x \geq 10) = 1 - P(x < 10) = 1 - P(x \leq 9)$$

$$= 1 - \text{binomcdf}(20, 0.40, 9) \approx \boxed{0.2447}$$

   (c) In the sample of size 20, what would be an unusually large number of households that have at least one dog?

$$np + 2\sqrt{npq} \approx 12.38$$

MORE THAN 12 IS UNUSUAL.

8. (8 points) For cars traveling at 30 miles per hour (mph), the distance required to brake to a stop is normally distributed with mean 50 ft and standard deviation 8 ft.

   (a) What is the probability that a car can brake to a stop in less than 38 ft?

$$P(x < 38) = \text{normalcdf}(-99999, 38, 50, 8) \approx \boxed{0.0668}$$

   (b) Within what distance can 90% of all cars brake to a stop?

$$\text{invNorm}(0.90, 50, 8) \approx \boxed{60.25 \text{ FT}}$$

9. (10 points) Suppose college professors at two-year institutions earn an average of \$65,608 per year with a standard deviation of \$4000. A sample of 100 two-year-college professors is randomly selected.

(a) What is the probability the the sample mean is greater than \$67,000?

$\mu_{\bar{x}} = 65608$ $\quad P(\bar{x} > 67000) = \quad \approx 0.00025$

$\sigma_{\bar{x}} = \frac{4000}{\sqrt{100}} = 400$ $\quad$ normalcdf(67000, 999999, 65608, 400)

(b) If your random sample actually produced a sample mean of \$67,000, would you consider that unusual? Explain.

YES! SUCH A SAMPLE MEAN IS VERY UNLIKELY---

MUCH LESS THAN 5%

(c) If the sample size was increased to 400, what would happen to your probability in part (a)? Why?

IT WOULD GET MUCH SMALLER---

THE SAMPLING DISTRIBUTION WILL

HAVE MUCH LESS SPREAD.

10. (12 points) A survey of 36 adults found that the mean age of a person's primary vehicle is 6.3 years. Assume that the standard deviation of the population is 0.9 years.

(a) Is the underlying sampling distribution normal or t? How do you know?

σ IS KNOWN

(b) Construct a 95% confidence interval estimate for the mean age of all primary vehicles.

Z Interval $\quad \bar{x} = 6.3$ $\quad (6.006, 6.594)$

Stats $\quad n = 36$ $\quad$ WE CAN BE 95% CONFIDENT

$\sigma = 0.9$ $\quad$ C-Level = 0.95 $\quad$ THAT THE TRUE MEAN IS IN OUR INTERVAL.

(c) Determine the sample size required to have a margin of error of ± 0.02 at the level $\alpha = 0.01$.

$z_{\alpha/2} = \text{invNorm}(0.995) \approx 2.576$

$$n \approx \left(\frac{2.576(0.9)}{0.02}\right)^2 \approx 13438$$

11. (10 points) A poison control center receives 1230 calls in a 30-day period.

(a) What is the mean number of calls per day?

$$\mu = \frac{1230}{30} = \boxed{41}$$

(b) What is the probability of the center receiving more than 49 calls on any given day?

Poisson

$\mu = 41$

$$P(x > 49) = 1 - P(x \le 49)$$

$$= 1 - \text{poissoncdf}(41, 49) \approx \boxed{0.0950}$$

(c) What is an unusually small number of daily calls?

$$\mu - 2\sqrt{\mu} \approx 28.19$$

28 OR FEWER ARE UNUSUAL

12. (8 points) In a survey of 3110 U.S. adults, 1435 say they have started paying some bills online. Find a 90% confidence interval estimate for the true population proportion. Give a one-sentence interpretation of your result.

1-Prop ZInt

$X = 1435$

$n = 3110$

C-Level = 0.90

(0.44671, 0.47612)

WE ARE 90% CONFIDENT THAT THE TRUE PROPORTION OF ADULTS PAYING BILLS ONLINE IS BETWEEN 44.7% AND 47.6%

13. (16 points) A large university reports that the mean salary of parents of an entering class is \$91,600. The university president randomly selects 28 families, and she finds the mean salary to be \$88,500 with a sample standard deviation of of \$9,915. Use the president's sample to test the university's reported claim at the level $\alpha = 0.10$.

(a) State the null and alternative hypotheses.

$H_0: \mu = 91600$

$H_1: \mu \neq 91600$

(b) Is the underlying sampling distribution normal or t? How do you know?

SAMPLING DIST. IS t SINCE σ IS NOT KNOWN.

(WE'LL ASSUME THE BACKGROUND DIST IS APPROXIMATELY NORMAL.)

(c) Compute the test statistic.

T-Test

Stats

$\mu_0 = 91600$

$\bar{X} = 88500$

$S_x = 9915$

$n = 28$

$\boxed{t \approx -1.654}$

(d) Find the P-value and draw a conclusion about the university's claim.

$\boxed{\text{P-VALUE} \approx 0.1096}$

SINCE P-VALUE $> \alpha$,

WE DO NOT REJECT H_0.

THERE IS NOT SUFFICIENT EVIDENCE TO REJECT THE UNIVERSITY CLAIM.