$\qquad$

Show all work to receive full credit. Supply explanations where necessary.

1. (14 points) Let $f(x)=2 x^{3}-3 x^{2}-36 x+10$. Find open intervals on which $f$ is increasing/decreasing. Then find and classify all local (relative) extreme values.
2. ( 8 points) In solving an optimization problem, Joe found that $x=1$ is a critical number of the function $P(x)=2 x+\frac{2}{x}$. Use calculus to show that Joe's critical number minimizes $P$.
3. (6 points) The graph of $f$ is shown below. For each part of this problem, find a labeled point that satisfies the given condition.

(a) $f^{\prime \prime}(x)=0$
(b) $f^{\prime}(x)=0$
(c) $f^{\prime \prime}(x)<0$
(d) $f(x)<0$
(e) $f^{\prime}(x)>0$
(f) $f^{\prime \prime}(x)>0$
4. (6 points) When the production level is 5000 units, marginal revenue is $\$ 5.15$ per unit and marginal cost is $\$ 5.75$ per unit. Do you expect maximum profit to occur at a production level above or below 5000 units? Explain your reasoning.
5. (8 points) Find the inflection point(s) of the graph of $g(x)=x e^{-x}$.
6. (8 points) Find the global (absolute) extreme values of $f(x)=2 x^{3}-9 x^{2}+12 x$ on the interval $-0.5 \leq x \leq 3$.
7. (6 points) Suppose $f$ has a continuous derivative whose values are given in the table below.

| $x$ | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $f^{\prime}(x)$ | 6 | 2 | 1 | -1 | -4 | -5 | -2 | 1 | 3 | 2 | -1 |

(a) Find reasonable estimates for the critical numbers of $f$.
(b) Determine whether each one of your critical numbers gives a local (relative) minimum or maximum. Briefly explain how you know.
8. (8 points) The revenue from selling $q$ items is $R(q)=800 q-q^{2}$, and the total cost is $C(q)=150+12 q$. Find the quantity that maximizes profit.
9. (8 points) The velocity, $v$, of an object at time $t$ is described in the table below.

| $t(\mathrm{sec})$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v(\mathrm{ft} / \mathrm{sec})$ | 3 | 6 | 10 | 16 | 22 | 20 | 18 |

(a) Use a right sum with $\Delta t=2$ to estimate the total distance traveled by the object.
(b) Use a left sum with $\Delta t=1$ to estimate the total distance traveled by the object.
(c) Which of your approximations do you think better estimates the distance traveled? Why?
10. (8 points) Use a left sum with 4 subintervals (rectangles) of equal width to estimate $\int_{0}^{1} e^{-x^{2}} d x$.
11. (5 points extra credit) The quantity of a drug in the bloodstream $t$ hours after a tablet is swallowed is given, in milligrams, by

$$
Q(t)=25\left(e^{-t}-e^{-2 t}\right) .
$$

What is the maximum quantity of the drug in the bloodstream?
12. (5 points extra credit) Sketch the graph of $f(x)=x+2$ over the interval from $x=0$ to $x=3$. Then use area to compute the exact value of $\int_{0}^{3}(x+2) d x$. Show all work or explain your reasoning.
$\qquad$
Score

Show all work to receive full credit. Supply explanations where necessary. You must work INDIVIDUALLY ON THIS EXAM.

1. (10 points) A landscape architect plans to enclose a 4000 square-foot rectangular region in a botanical garden. She will use shrubs costing $\$ 30$ per foot along three sides and flowers costing $\$ 10$ per foot along the fourth side. Determine a function giving the total cost of the project and then find the minimum cost.
2. ( 7 points) Use a left or right sum with 8 subintervals (rectangles) of equal width to estimate $\int_{0}^{2} \frac{1}{1+t^{2}} d t$. Then use your calculator to estimate the value of the definite integral.
3. (3 points) The graph of $f$ is shown below. Find a reasonable estimate for $\int_{0}^{7} f(x) d x$.

