Math 157 - Quiz 2

August 31, 2016

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY.

1. (2 points) Consider the graph of the function $f(x) = \dfrac{2^x + 6}{2}$. Find the slope of the secant line that passes through the points where $x = 2$ and $x = 3$.

$$f(2) = \frac{2^2 + 6}{2} = 5$$

$$f(3) = \frac{2^3 + 6}{2} = 7$$

$$m = \frac{f(3) - f(2)}{3 - 2} = \frac{7 - 5}{1} = \boxed{2}$$

2. (3 points) The following functions describe the populations (in thousands of people) of three different towns at time t (in years).

$(A) {-}{-}{-} P(t) = 17(1.5^t)$ $\qquad$ $(B) {-}{-}{-} P(t) = 120(1.01^t)$ $\qquad$ $(C) {-}{-}{-} P(t) = 42(0.97^t)$

Answer each question and explain how you know.

(a) Which town is growing the fastest?

$\left(A\right)$ -- $a = 1.5 > 0$ & THIS IS gREATEST
FOR THE THREE TOWNS

(b) Which town is initially the largest?

$\left(B\right)$ -- $P_0 = 120$ & THIS IS THE LARGEST
INITIAL POPULATION

(c) Which town has a population that is decreasing?

$\left(C\right)$ -- $a = 0.97 < 1$

SINCE BASE IS LESS THAN 1,
$P(t)$ IS DECREASING

3. (3 points) $P = 140$ when $t = 3$ and $P = 100$ when $t = 1$. Find the values of the parameters k and P_0 so that $P(t) = P_0 e^{kt}$.

$P = 140$ WHEN $t = 3 \implies 140 = P_0 e^{3k}$

$P = 100$ WHEN $t = 1 \implies 100 = P_0 e^{k}$

$$\frac{140}{100} = \frac{P_0 e^{3k}}{P_0 e^{k}} \implies 1.4 = e^{2k}$$

$$\ln 1.4 = 2k$$

$$\boxed{\frac{\ln 1.4}{2} = k \approx 0.168}$$

$$100 = P_0 e^{\frac{\ln 1.4}{2}}$$

$$\implies \boxed{P_0 = \frac{100}{e^{\frac{\ln 1.4}{2}}} \approx 84.515}$$

4. (2 points) A quantity is growing according to the formula $P(t) = 54e^{0.2t}$. Rewrite this as an equivalent function in the form $P(t) = P_0 a^t$ and determine the growth rate per unit time.

$$a = e^{0.2} \approx 1.2214$$

$$P(t) = 54 \left(1.2214 \right)^{t}$$

$$\Downarrow$$

RATE OF growth IS $\approx 22\%$

PER UNIT TIME.