

Math 157 - Quiz X

November 16, 2016

Name key

Score _____

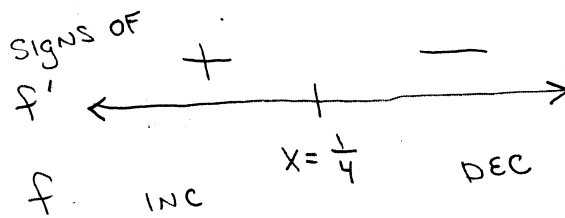
This is an optional quiz. The score on this quiz will replace your current lowest quiz score (if that helps you). It is due Wednesday, November 23. Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find the absolute (global) maximum value of $f(x) = 2xe^{-4x}$. Explain how you know you've found a global max.

$$f'(x) = 2e^{-4x} - 8xe^{-4x}$$

$$= 2e^{-4x}(1 - 4x) = 0$$

$$\Rightarrow x = \frac{1}{4}$$



$$f\left(\frac{1}{4}\right) = \frac{2}{4}e^{-1} = \frac{1}{2e} \approx 0.184$$

THIS MUST BE A GLOBAL MAX BECAUSE $x = \frac{1}{4}$ IS THE ONLY CRIT PT.

f INCREASES TO $x = \frac{1}{4}$, THEN DECREASES.

2. (4 points) Find the points of inflection of the graph of $f(x) = \frac{x}{x^2+1}$.

(There are three.)

$$f'(x) = \frac{x^2+1 - 2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

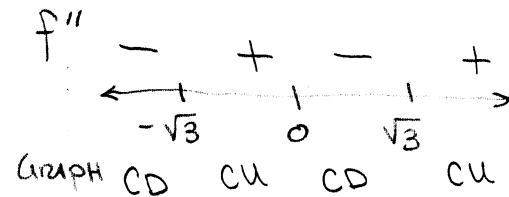
$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{-2x(x^2+1) - (1-x^2)(4x)}{(x^2+1)^3}$$

$$= \frac{2x^3 - 6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3} = 0$$

$$\Rightarrow x=0, x=\sqrt{3}, x=-\sqrt{3}$$

SIGNS OF f''



CHANGES CONCAVITY AT EACH PT.

INF POINTS ARE

$$\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right), (0, 0),$$

$$\text{AND } \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$$

3. (2 points) Section 4.3, Page 193, Problem #38

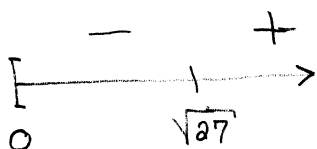
a) $D = \text{TOTAL DISTANCE} = \# \text{ OF DROPS} \times \text{HEIGHT PER DROP}$

$$= n(x) \cdot x$$
$$= x + \frac{27}{x}$$

b) MINIMIZE D...

$$D' = 1 - \frac{27}{x^2} = 0 \text{ WHEN } x^2 = 27$$

$$x = \sqrt{27}$$



GLOBAL MIN

TOTAL DISTANCE IS A MIN

WHEN $x = \sqrt{27} \approx 5.2 \text{ m}$