Math 157 - Test 1

September 21, 2016

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Consider the data given in the following table.

x	1	2	4	7	10
y	5	2	-4	-13	-20

(a) Could this data be representative of linear function? Show work.

$$-3 = \frac{2-5}{2-1} = \frac{-4-2}{4-2} = \frac{-13-(-4)}{7-4} \neq \frac{-20-(-13)}{10-7} = \frac{-7}{3}$$

THE DATA DOES NOT REPRESENT A LINEAR FUNCTION.

(b) Find an equation of the line that passes through the first two points, $(1, 5)$ and $(2, 2)$.

$$m = \frac{2-5}{2-1} = -3$$

$$y - 2 = -3(x-2)$$

$$y - 2 = -3x + 6$$

$$\boxed{y = -3x + 8}$$

2. (9 points) An antique worth \$79 today is gaining value at 12.5% per year.

(a) Determine the function of the form $P(t) = P_0 a^t$ that gives the value of the antique after t years.

$P_0 = 79 \quad a = 1.125$

$$\boxed{P(t) = 79\,(1.125^t)}$$

(b) What will be the value of the antique in 8.4 years?

$$P(8.4) = 79\,(1.125^{8.4}) \approx \boxed{\$\,212.48}$$

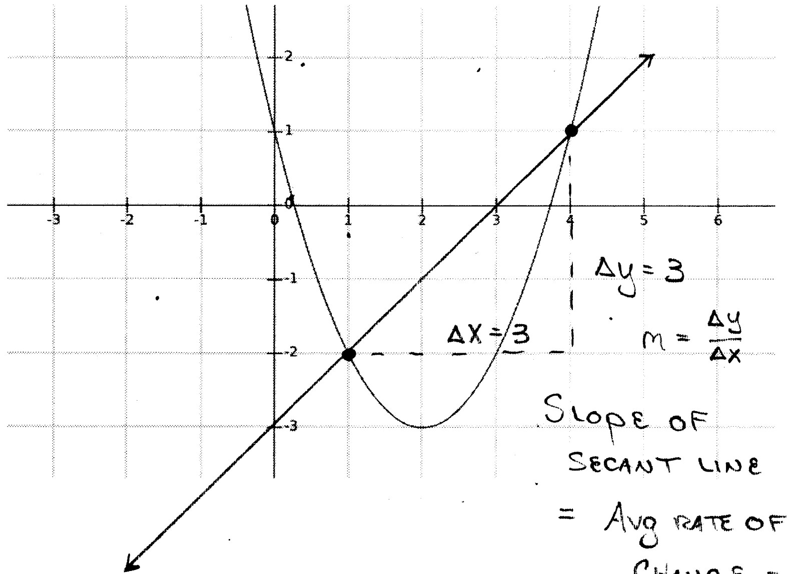
(c) Rewrite the function in the form $P(t) = P_0 e^{kt}$. (Determine P_0 and k.)

$P_0 = 79$

$e^k = 1.125 \Rightarrow k = \ln(1.125) \approx 0.1178$

$$\boxed{P(t) = 79\,e^{0.1178t}}$$

3. (8 points) The graph of the function $f(x) = x^2 - 4x + 1$ is shown below. Find the average rate of change of f over the interval from $x = 1$ to $x = 4$. Then illustrate your answer on the graph.



Slope of secant line = Avg rate of change = 1

$$\frac{f(4)-f(1)}{4-1} = \frac{1-(-2)}{4-1} = \frac{3}{3} = \boxed{1}$$

4. (6 points) $P = 80$ when $t = 5$ and $P = 30$ when $t = 2$. Find the values of the parameters k and P_0 so that $P(t) = P_0e^{kt}$.

$$80 = P_0 e^{5k}$$

$$30 = P_0 e^{2k} \Rightarrow P_0 = \frac{30}{e^{2k}} \Rightarrow \underbrace{80 = \frac{30}{e^{2k}}\left(e^{5k}\right) = 30e^{3k}}$$

$$\frac{8}{3} = e^{3k} \Rightarrow k = \frac{\ln 8/3}{3} \approx 0.3269$$

$$P_0 = \frac{30}{e^{2k}} = 15.60$$

5. (6 points) Solve for t.

(a) $e^{3t} = 100$

$$3t = \ln 100$$

$$t = \frac{\ln 100}{3} \approx 1.535$$

(b) $58 = 17 \cdot 4^t$

$$\frac{58}{17} = 4^t$$

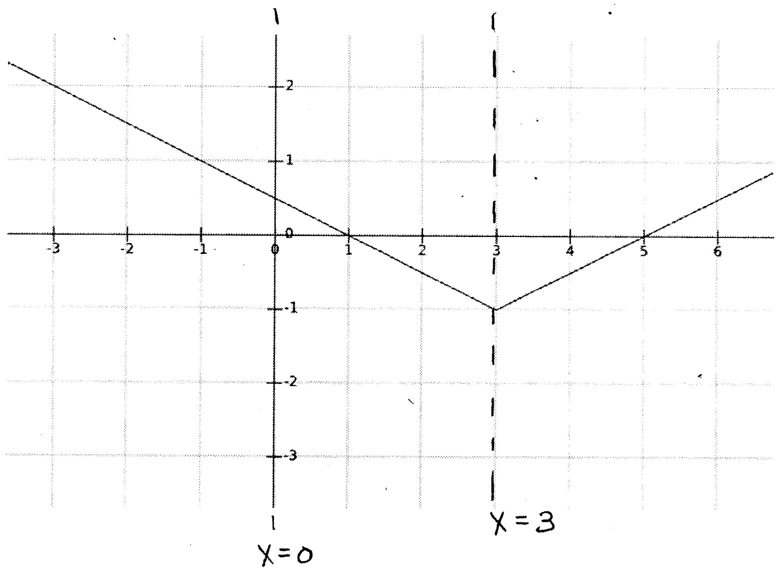
$$\ln \frac{58}{17} = \ln 4^t = t \ln 4$$

$$t = \frac{\ln 58/17}{\ln 4} \approx 0.885$$

6. (5 points) Determine the relative rate of change of $g(x) = 10e^{0.25x}$ over the interval from $x = 1$ to $x = 2$.

$$\frac{g(2)-g(1)}{g(1)\,[2-1]} = \frac{10e^{0.5} - 10e^{0.25}}{10e^{0.25}} \approx \boxed{0.284}$$

7. (6 points) The graph of the function f is shown below.



Determine each limit or explain why it does not exist.

(a) $\lim_{x \to 3} f(x) = -1$ (As x gets close to 3, y values get close to -1)

(b) $\lim_{x \to 0} f(x) = \frac{1}{2}$ (As x gets close to 0, y values get close to $\approx \frac{1}{2}$)

8. (6 points) Use algebra to find the limit.

$\frac{0}{0}$ NONSENSE

$$\lim_{x\to 0} \frac{(x-5)^2 - 25}{4x}$$

$$= \lim_{x\to 0} \frac{x^2 - 10x + 25 - 25}{4x} = \lim_{x\to 0} \frac{x(x-10)}{4x}$$

$$= \lim_{x\to 0} \frac{x-10}{4} = -\frac{10}{4} = -2.5$$

9. (8 points) Let $g(x) = \ln x$. Use at least four small intervals to estimate $g'(5)$.

INTERVAL	RATE OF CHANGE	
[4.9, 5.1]	$\frac{\ln 5.1 - \ln 4.9}{0.2}$	≈ 0.20003
[4.99, 5.01]	$\frac{\ln 5.01 - \ln 4.99}{0.02}$	≈ 0.20000
[5, 5.001]	$\frac{\ln 5.001 - \ln 5}{0.001}$	≈ 0.19998
[4.999, 5]	$\frac{\ln 5 - \ln 4.999}{0.001}$	≈ 0.20002

IT LOOKS LIKE

$g'(5) = 0.2$

10. (5 points) The graph of the function f passes through the point $(2, 7)$, and at that point, $f'(2) = -5$. Find an equation for the tangent line at the point $(2, 7)$.

$\underbrace{f'(2) = -5}$

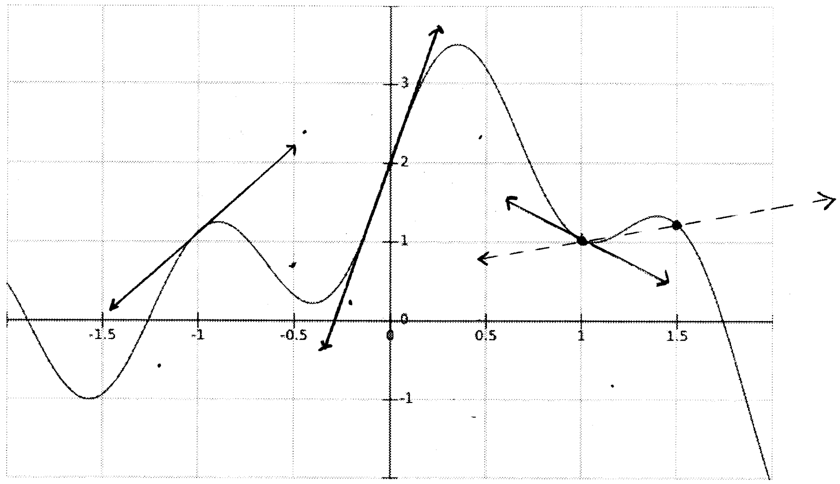
$m = -5$

POINT $(2,7)$ $\longrightarrow$ $y - 7 = -5(x-2)$

$y - 7 = -5x + 10$

$y = -5x + 17$

11. (10 points) The graph of the function f is shown below.



(a) If $f(1)$ positive or negative? How do you know?

THE GRAPH AT $X=1$ LIES ABOVE THE X-AXIS. IN FACT $f(1) \approx 1$

(b) If $f'(1)$ positive or negative? How do you know?

TANGENT LINE AT $X=1$ HAS NEGATIVE SLOPE.

(c) Which is greater $f'(-1)$ or $f'(0)$? Explain your reasoning.

$f'(0) > f'(-1)$ --- THE SLOPE OF THE TANGENT LINE AT $X=0$ IS GREATER THAN THAT AT $X=-1$

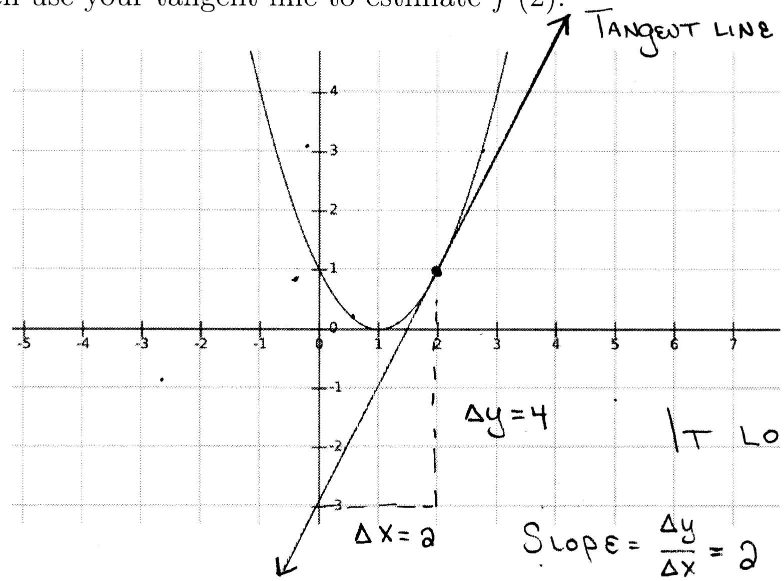
(d) At the point where $x = 1.5$, is the graph concave down or concave up?

CURVES DOWNWARD (SPILLS WATER)

(e) Is the value of $\dfrac{f(1.5)-f(1)}{1.5-1}$ positive, negative, or zero? How do you know?

THE SLOPE OF THE SECANT LINE THROUGH THE POINTS WHERE $X=1.5$ & $X=1$ IS POSITIVE. (SEE DASHED LINE ABOVE.)

12. (5 points) The graph of the function f is shown below. Sketch the tangent line at $x = 2$. Then use your tangent line to estimate $f'(2)$.



$\text{Slope} = \frac{\Delta y}{\Delta x} = 2$

IT LOOKS LIKE $f'(2) = 2$

13. (6 points) Annual sales of music CDs have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.

(a) Find a formula for the annual sales, S, in millions of CDs, as a linear function of the number of years, t, since 2000.

$t = 0, \; S = 942.5$

$t = 8, \; S = 384.7$

$$m = \frac{\Delta S}{\Delta t} = \frac{384.7 - 942.5}{8 - 0} = -69.725$$

$$S = -69.725t + 942.5$$

(b) Use your function to predict CD sales in 2017.

$$t = 17 \Rightarrow S = -69.725(17) + 942.5$$

$$= \underbrace{-242.825}_{\text{A NEGATIVE \# OF CDs IS NOT POSSIBLE!}}$$

(c) Solve the equation $S(t) = 0$. What is the significance of your solution? (Use units when answering.)

$$-69.725t + 942.5 = 0$$

$$\Rightarrow t = \frac{942.5}{69.725} \approx 13.5 \text{ yrs}$$

AFTER ABOUT 13.5 YEARS, SALES WILL DROP TO ZERO. (THAT'S WHY THE ANSWER ABOVE IS NEGATIVE!)

14. (4 points) The table shown below gives values of the function h at selected points. Use the table to find a reasonable estimate for $\lim_{x \to 2} h(x)$.

x	$h(x)$
1.9	8.4573
1.99	8.4092
1.999	8.4001
2.1	8.3751
2.01	8.3952
2.001	8.3998

AS X GETS CLOSER AND CLOSER TO 2, h(x) GETS CLOSER AND CLOSER TO 8.4

LOOKS LIKE $\lim_{x \to 2} h(x) = 8.4$

15. (4 points) The table shown below gives values of the function h at selected points. Explain why this table cannot be used to estimate $\lim_{x \to 0} h(x)$.

x	$h(x)$
-3	17.7
-2	13.4
-1	10.1
0	9.3
1	10.8
2	14.6

THE X-VALUES IN THE TABLE ARE NOT SUFFICIENTLY CLOSE TO X=0 FOR US TO BE ABLE TO DRAW A CONCLUSION.

16. (4 points) The table shown below gives values of the function h at selected points. Use the table to find a reasonable estimate for $\lim_{x \to 1} h(x)$.

x	$h(x)$
0.9	3.4619
0.99	3.9012
0.999	3.9994
1.1	-3.3751
1.01	-3.8976
1.001	-3.9989

IT LOOKS LIKE $\lim_{x \to 1} h(x)$ DNE.

AS X GETS CLOSE TO X=1, THE VALUES OF h(x) SEEM TO GET CLOSER TO 2 DIFFERENT NUMBERS (MAYBE 4 & -4).