Math 157 - Test 2
October 19, 2016

Name Key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) The function $p(x) = 0.07x^3 - 0.75x^2 + 1.90x + 3.10$ gives the profit in dollars from the sale of x items.

   (a) Compute $p(4)$ and $p'(4)$. Using units, explain what each of these values represents.

$p(4) = 0.07(4)^3 - 0.75(4)^2 + 1.90(4) + 3.10 = 3.18$

$p(4) =$ \$3.18 = PROFIT FROM SELLING 4 ITEMS

$p'(x) = 0.21x^2 - 1.5x + 1.90$

$p'(4) = 0.21(4)^2 - 1.5(4) + 1.90 = -0.74$

$p'(4) = -0.74$ DOLLARS/ITEM AT THE SALES LEVEL OF 4 ITEMS, YOU ARE LOSING 74¢ PER ITEM

   (b) Based on your results in part (a), should you sell more or fewer items to increase profit? Explain.

SELL FEWER ITEMS --- $p'(4)$ IS NEGATIVE SO YOUR PROFIT IS DECREASING. SELLING MORE ITEMS WILL CAUSE YOUR PROFIT TO DECREASE EVEN MORE.

2. (8 points) For the function $f(x)$, we know that $f(20) = 68$ and $f'(20) = -3$. Find an equation of the line tangent to the graph of f at the point where $x = 20$. Then use your tangent line equation to approximate $f(19)$.

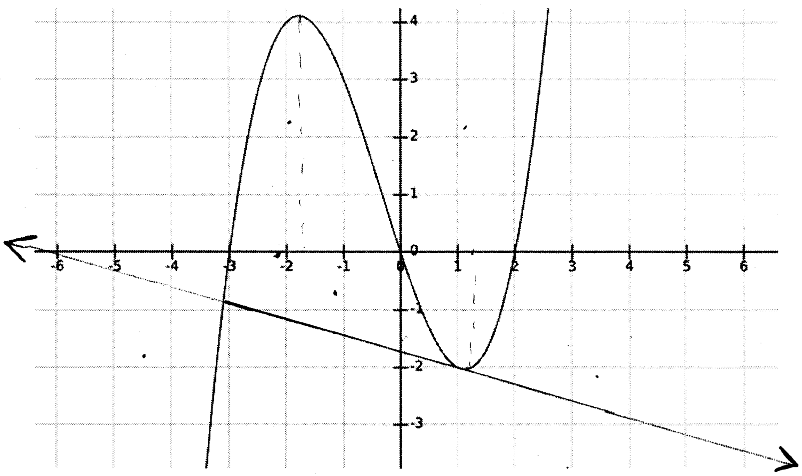
$y - 68 = -3(x - 20)$

OR

$y = -3x + 128$

$f(19) \approx -3(19) + 128 = 71$

3. (10 points) The graph of the function $f(x)$ is shown below.



(a) Use the graph to estimate $f'(1)$.

$\boxed{f'(1) \approx -\frac{2}{7}}$

Tangent line at $x = 1$

$m \approx \frac{-1}{3.5} = -\frac{2}{7}$

(b) Find an equation of the line tangent to the graph of f at $x = 1$. (Because it's based on the graph, your line will only be an estimate.)

y-intercept is about $(0, -1.75)$

Slope is about $-\frac{2}{7}$

$\boxed{y = -\frac{2}{7}x - 1.75}$

(c) Estimate the critical points of f.

Horizontal tangent lines at interior points

$\boxed{x \approx 1.2 \text{ and } x \approx -1.8}$

4. (8 points) Find the critical points of $g(x) = (x^2 - 4x)^2$.

$g'(x) = 2(x^2-4x)(2x-4) = 4x(x-4)(x-2)$

$g'(x) = 0 \Rightarrow \boxed{x = 0,\ x = 4,\ x = 2}$

These are all domain interior pts where $g'(x) = 0$. $g'(x)$ DNE nowhere.

5. (9 points) For each part of this problem, a function, its domain, and a point are given. Determine whether the point is a critical point. Explain your answer.

   (a) $f(x) = x^2$, Domain: $[0, 4]$, Point: $x = 0$

   CAN'T BE A CRIT PT.
   IT IS A DOMAIN ENDPOINT,
   NOT INTERIOR PT.

   (b) $g(x) = x^2 + 6x$, Domain: $[-5, 10]$, Point: $x = 0$

   $g'(x) = 2x + 6$

   $g'(0) = 6$

   $X = 0$ IS NOT A CRIT PT BECAUSE NEITHER $g'(0) = 0$ NOR $g'(0)$ DNE.

   (c) $h(x) = 2x^3 + 3x^2 - 12x + 7$, Domain: All real numbers, Point: $x = -2$

   $h'(x) = 6x^2 + 6x - 12$

   $h'(-2) = 24 - 12 - 12 = 0$

   YES! $X = -2$ IS A CRIT PT --- DOMAIN INTERIOR PT WHERE $h'(x) = 0$.

6. (7 points) With t in years since January 1, 2010, the population P of Slim Chance is predicted by

$$P = 35000(0.98)^t.$$

At what rate will the population be changing on January 1, 2023? Give units with your answer.

$$\frac{dP}{dt} = 35000(\ln 0.98)(0.98)^t$$

$$\left.\frac{dP}{dt}\right|_{t=13} = 35000(\ln 0.98)(0.98)^{13} \approx -543.8 \text{ PEOPLE PER YEAR}$$

7. (20 points) Determine the derivative of each function.

(a) $h(t) = \frac{5}{t} + \frac{8}{t^2}$ $= 5t^{-1} + 8t^{-2}$

$$\boxed{h'(t) = -5t^{-2} - 16t^{-3}}$$

(b) $f(x) = 1.8e^{2x} - 4.7\ln x$

$$\boxed{f'(x) = 3.6e^{2x} - \frac{4.7}{x}}$$

(c) $Q = \ln(\ln t)$

$$\frac{dQ}{dt} = \frac{1}{\ln t}\left(\frac{1}{t}\right) = \boxed{\frac{1}{t \ln t}}$$

(d) $w = \frac{7y - 8y^2}{6 + 2y}$

$$\boxed{\frac{dw}{dy} = \frac{(6+2y)(7-16y) - (7y-8y^2)(2)}{(6+2y)^2}}$$

(e) $g(z) = \sqrt{z}\,e^{-z}$ $= z^{1/2}e^{-z}$

$$\boxed{g'(z) = \frac{1}{2}z^{-1/2}e^{-z} + z^{1/2}(-1)e^{-z}}$$

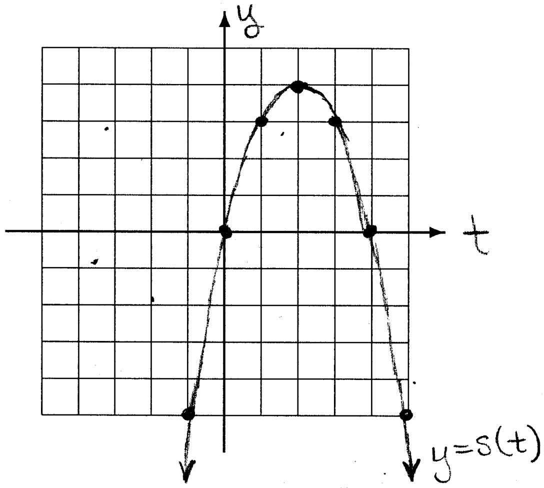
8. (8 points) Sketch the graph of the function $s(t) = 4t - t^2$.

$s(t) = t(4-t)$

ZEROS ARE $t=0$, $t=4$

VERTEX AT $t=2$, $s(2)=4$

PARABOLA OPENS DOWN



Use your graph to determine whether each of the following is positive, negative, or zero. Give a very brief explanation for each answer.

$s'(0)$

POSITIVE - SLOPE AT $t=0$ IS POSITIVE

$s'(3)$

NEGATIVE - SLOPE AT $t=3$ IS NEGATIVE

$s'(2)$

ZERO - TANGENT LINE AT $t=2$ IS HORIZONTAL

9. (7 points) Given that $f(1) = 9$ and $f'(1) = 24$, find $g'(1)$ if $g(x) = \sqrt{f(x)}$.

$$g(x) = [f(x)]^{1/2}$$

$$g'(x) = \frac{1}{2}[f(x)]^{-1/2} f'(x)$$

$$g'(1) = \frac{1}{2}[f(1)]^{-1/2} f'(1) = \frac{1}{2}(9)^{-1/2}(24)$$

$$= 4$$

10. (7 points) Find the second derivative of the function $h(x) = x^4 + e^{-3x} + \ln x$.

$$h'(x) = 4x^3 - 3e^{-3x} + \frac{1}{x}$$ ← or x^{-1}

$$h''(x) = 12x^2 + 9e^{-3x} - x^{-2}$$

11. (8 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	-2	-1	2
$f(x)$	1	3	-2
$f'(x)$	2	-1	-1
$g(x)$	2	0	-2
$g'(x)$	-3	-2	1

(a) If $h(x) = f(x) \cdot g(x)$, use the product rule to compute $h'(-1)$.

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(-1) = (-1)(0) + (3)(-2) = -6$$

(b) If $h(x) = \dfrac{f(x)}{g(x)}$, use the quotient rule to compute $h'(2)$.

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{(-2)(-1) - (-2)(1)}{(-2)^2} = \frac{4}{4} = \boxed{1}$$