

Math 157 - Test 3a
November 23, 2016

Name key Score _____

Show all work. Supply explanations where necessary.

1. (8 points) Let $g(x) = x^3 - 6x + 1$. Find the critical points of g . Then use the 2nd derivative to determine whether there is a relative (local) maximum or minimum at each critical point.

$$g'(x) = 3x^2 - 6$$

$g'(x)$ DNE NOWHERE

$$g'(x) = 0$$

$$\Rightarrow 3x^2 - 6 = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$g''(x) = 6x$$

$$g''(\sqrt{2}) = 6\sqrt{2} > 0 \quad \text{CU AT } x = \sqrt{2}$$

\Rightarrow REL MIN AT $x = \sqrt{2}$

$$g''(-\sqrt{2}) = -6\sqrt{2} < 0 \quad \text{CD AT } x = -\sqrt{2}$$

\Rightarrow REL MAX AT $x = -\sqrt{2}$

2. (6 points) Consider the function $f(x) = x^3 + 3x^2 - 9x - 10$ restricted to the domain $[-2, 2]$. Find the absolute (global) extreme values.

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ &= 3(x^2 + 2x - 3) \\ &= 3(x+3)(x-1) \end{aligned}$$

$f'(x)$ DNE NOWHERE ON $[-2, 2]$

$$f'(x) = 0 \Rightarrow \cancel{x = -3} \text{ or } x = 1$$

ENDPTS $x = -2$ or $x = 2$

$$f(-2) = 12 \leftarrow \text{ABS MAX}$$

$$f(1) = -15 \leftarrow \text{ABS MIN}$$

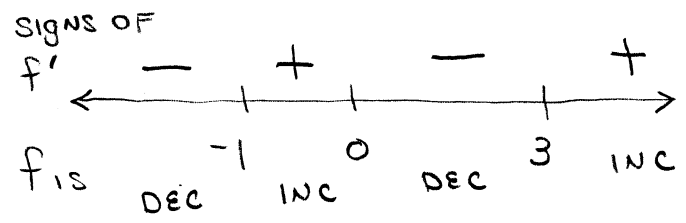
$$f(2) = -8$$

POLYNOMIAL -- NICE EVERYWHERE

3. (12 points) Let $f(x) = 3x^4 - 8x^3 - 18x^2 + 6$.

(a) Find open intervals on which f is increasing/decreasing.

$$\begin{aligned} f'(x) &= 12x^3 - 24x^2 - 36x \\ &= 12x(x^2 - 2x - 3) \\ &= 12x(x-3)(x+1) = 0 \\ x &= 0, x = 3, x = -1 \end{aligned}$$



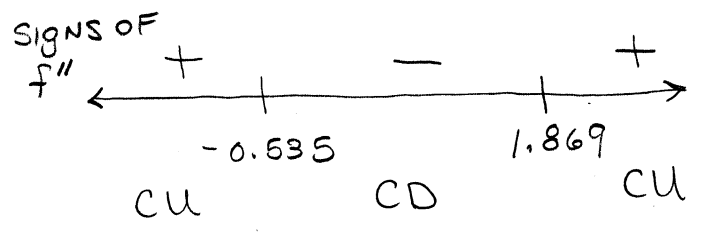
f is DECREASING ON $(-\infty, -1) \cup (0, 3)$ AND INCREASING ON $(-1, 0) \cup (3, \infty)$

(b) Identify all relative (local) extreme values.

$f(-1) = -1$ IS A REL MIN
 $f(0) = 6$ IS A REL MAX
 $f(3) = -129$ IS A REL MIN

(c) Find open intervals on which the graph of f is concave up/down.

$$\begin{aligned} f''(x) &= 36x^2 - 48x - 36 \\ &= 12(3x^2 - 4x - 3) = 0 \\ \Rightarrow x &= \frac{4 \pm \sqrt{16 + 36}}{6} \\ &\approx -0.535, 1.869 \end{aligned}$$



GRAPH IS CU ON $(-\infty, -0.535) \cup (1.869, \infty)$ AND CD ON $(-0.535, 1.869)$

(d) Find all points of inflection.

CONCAVITY CHANGES AT BOTH

INFLECTION PTS
 $x \approx -0.535, y \approx 2.317$
 AND
 $x \approx 1.869, y \approx -72.465$

4. (6 points) Suppose f has a continuous derivative whose values are given in the table below.

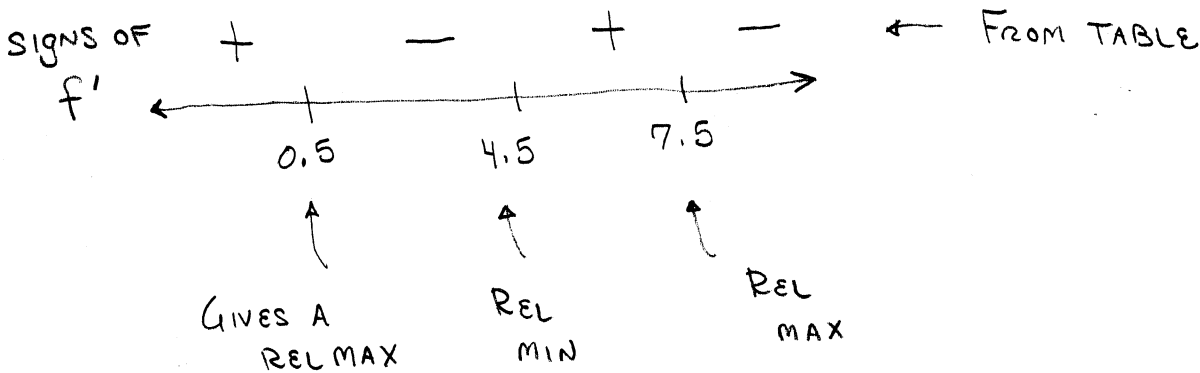
| | | | | | | | | | | | |
|---------|----|----|---|----|----|----|----|---|---|---|----|
| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $f'(x)$ | 8 | 2 | 1 | -2 | -7 | -5 | -1 | 1 | 4 | 2 | -1 |

- (a) Find reasonable estimates for the critical points of f .

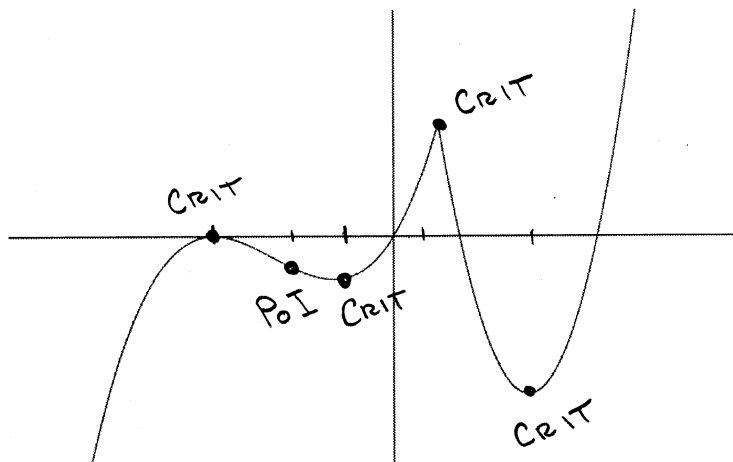
$f'(x) = 0$ BETWEEN $x=0$ AND $x=1$
 BETWEEN $x=4$ AND $x=5$
 AND BETWEEN $x=7$ AND $x=8$

LET'S SAY CRIT
 PTS ARE
 $x=0.5, x=4.5,$
 $x=7.5$

- (b) Determine whether each one of your critical numbers gives a local (relative) minimum or maximum. Briefly explain how you know (or show a sign chart).



5. (5 points) Identify any critical points and points of inflection on the graph below. (Say which are which!)



6. (6 points) Find the points of inflection of the graph of $f(x) = x^4 + x^3 - 3x^2 + 2$.

$$f'(x) = 4x^3 + 3x^2 - 6x$$

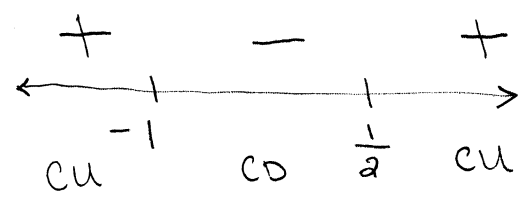
$$f''(x) = 12x^2 + 6x - 6$$

$$= 6(2x^2 + x - 1)$$

$$= 6(2x - 1)(x + 1) = 0$$

$$\Rightarrow x = \frac{1}{2}, x = -1$$

SIGNS OF f''



CONCAVITY CHANGES AT BOTH

$(-1, f(-1)) = (-1, -1)$
 AND $(\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, \frac{23}{16})$
 ARE BOTH INF PTS.

7. (8 points) The quantity of a drug in the bloodstream t hours after a tablet is swallowed is given by

$$q(t) = 40(e^{-t} - e^{-2t}),$$

where $q(t)$ is in milligrams. Find the maximum amount of drug in the blood. Explain how you know you found an absolute (global) maximum.

$$q'(t) = -40e^{-t} + 80e^{-2t} = 0$$

$$\Downarrow$$

$$80e^{-2t} = 40e^{-t}$$

$$\Downarrow$$

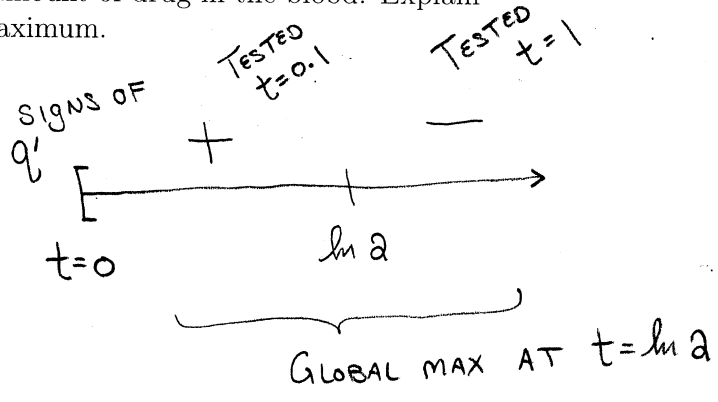
$$80 = 40e^t$$

$$\Downarrow$$

$$2 = e^t$$

$$\Downarrow$$

$$t = \ln 2$$



$$q(\ln 2) = 40\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= 10 \text{ mg}$$

8. (6 points) When the production level is 4000 units, marginal revenue is \$8.76 per unit and marginal cost is \$7.12 per unit. Do you expect maximum profit to occur at a production level above or below 4000 units? Explain your reasoning.

$$P'(x) = R'(x) - C'(x)$$

$$P'(4000) = 8.76 - 7.12$$

$$= 1.64 \Rightarrow P'(4000) \text{ is pos.}$$

$\Rightarrow P$ IS INCREASING AT $x = 4000$

MAX IS AT LEVEL ABOVE $x = 4000$

9. (10 points) An candy company finds that at a price of \$5.00, demand is 2000 units. For every \$0.25 decrease in price, demand increases by 400 units. Find the price and quantity sold that maximizes revenue.

DEMAND:

$$q = 2000, p = 5$$

$$m = \frac{\Delta p}{\Delta q} = \frac{-0.25}{400}$$

$$p - 5 = -\frac{0.25}{400}(q - 2000)$$

$$p = -\frac{0.25}{400}q + 6.25$$

REVENUE:

PRICE x QUANTITY

$$R(q) = -\frac{0.25}{400}q^2 + 6.25q$$

$$R'(q) = -\frac{0.50}{400}q + 6.25 = 0$$

$$\Rightarrow q = \frac{6.25(400)}{0.50} = 5000$$

$q = 5000$ IS A MAX
BECAUSE GRAPH OF R IS A
DOWNWARD OPENING PARABOLA
(AND WE FOUND VERTEX).

$$q = 5000 \Rightarrow p = \$3.13$$

10. (2 points) If a company's profit is increasing, what can you say about the company's marginal profit?

MARGINAL PROFIT = P' IS POSITIVE.

11. (5 points) Let $h(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$. Show that $h''(2) = 0$. Is there an inflection point at $x = 2$?

$$h'(x) = 4x^3 - 24x^2 + 48x - 32$$

$$h''(x) = 12x^2 - 48x + 48 = 12(x^2 - 4x + 4) = 12(x-2)^2$$

$$h''(2) = 12(4) - 48(2) + 48 = 0$$

No INFLECTION PT. CONCAVITY DOES NOT CHANGE.

signs of h''

12. (12 points) The velocity, v , of an object is sampled at various times, t , in seconds, throughout its motion.

| | | | | | | | |
|--------------|---|---|----|----|----|----|----|
| t (sec) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| v (ft/sec) | 5 | 9 | 12 | 13 | 22 | 20 | 15 |

- (a) Use a right sum with $\Delta t = 2$ to estimate the total distance traveled by the object.

$$12(2) + 22(2) + 15(2)$$

$$= 98 \text{ FT}$$

- (b) Use a left sum with $\Delta t = 1$ to estimate the total distance traveled by the object.

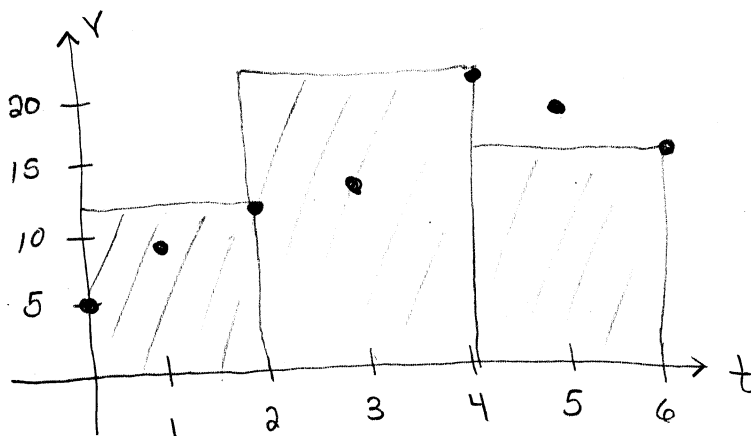
$$5(1) + 9(1) + 12(1) + 13(1) + 22(1) + 20(1)$$

$$= 81 \text{ FT}$$

- (c) Which of your approximations do you think better estimates the distance traveled? Why?

PROBABLY (b) BECAUSE IT USE SMALLER SUBINTERVALS ($\Delta t = 1$) AND MORE DATA.

- (d) Use rectangles to illustrate the right sum in part (a).



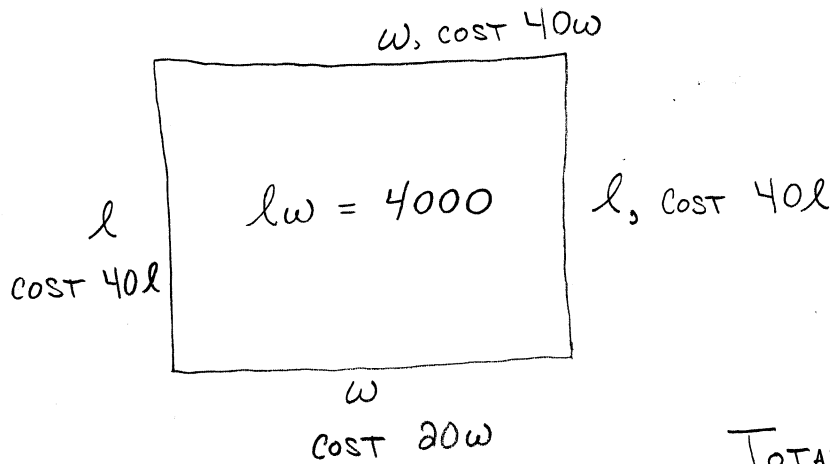
SHADED
AREA
= RIGHT
SUM

Math 157 - Test 3b
November 23, 2016

Name Key Score _____

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (8 points) A landscape architect plans to enclose a 4000 square-foot rectangular region in a botanical garden. She will use shrubs costing \$40 per foot along three sides and flowers costing \$20 per foot along the fourth side. Determine a function giving the total cost of the project and then find the minimum cost.



TOTAL COST

$$= C = 80l + 60w$$

$$l = \frac{4000}{w}$$

↓

$$C = \frac{80(4000)}{w} + 60w$$

$$C = \frac{320000}{w} + 60w$$

$$C' = -\frac{320000}{w^2} + 60 = 0$$

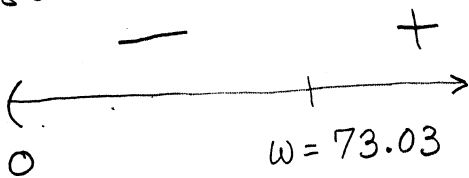
$$\frac{320000}{w^2} = 60$$

↓

$$w^2 = 5333.\bar{3}$$

$$w = 73.03$$

SIGNS OF
 C'



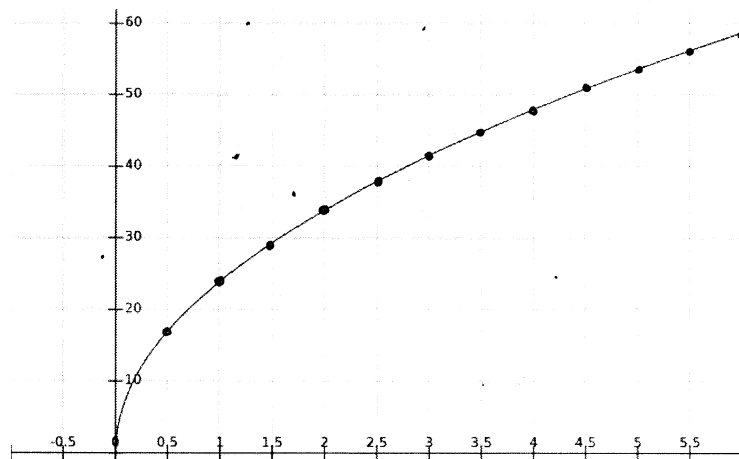
↑
GIVES A
GLOBAL MIN!

$$w \approx 73.03 \text{ FT}$$

$$\Rightarrow l \approx 54.77 \text{ FT}$$

$$\Rightarrow C \approx \$8763.40$$

2. (6 points) A motorcycle accelerates smoothly over 6 seconds. The graph of the motorcycle's velocity function is shown below. Estimate the total distance traveled by the motorcycle. (Time is measured in seconds and velocity in feet per second.)



I'll approximate with a right sum

using $\Delta x = 0.5$

$$\begin{aligned} \text{Right sum} &\approx 0.5 (17 + 24 + 29 + 34 + 38 + 41 + \\ &\quad 45 + 48 + 51 + 54 + 56 + 58) \\ &= \boxed{247.5 \text{ FT}} \end{aligned}$$

THIS APPEARS TO OVERESTIMATE

THE DISTANCE SINCE THE

VELOCITY FUNCTION IS INCREASING.