

Prove  $\frac{d}{dx} \cos x = -\sin x \dots$

$$\frac{d}{dx} \cos x = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \rightarrow 0} \left[ \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{\cos x \cos h - \cos x}{h} \right] - \lim_{h \rightarrow 0} \left( \frac{\sin x \sin h}{h} \right)$$

$$= (\cos x) \lim_{h \rightarrow 0} \left( \frac{\cos h - 1}{h} \right) - (\sin x) \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

*(Note: In the original image, arrows point from the limits to 0 and 1 respectively.)*

$$= (\cos x) (0) - (\sin x) (1)$$

$$= -\sin x$$