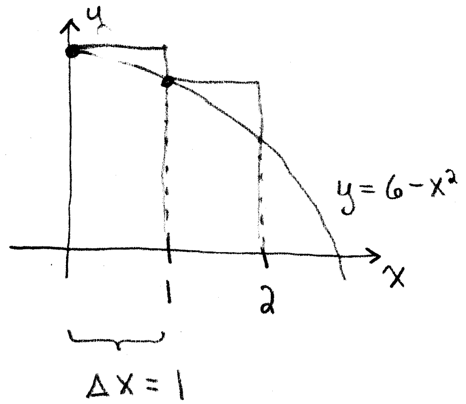
$f(x) = 6 - x^2$ ON $[0, 2]$

RIEMANN SUMS USING LEFT SUBINTERVAL ENDPOINTS...

$\boxed{N = 2}$

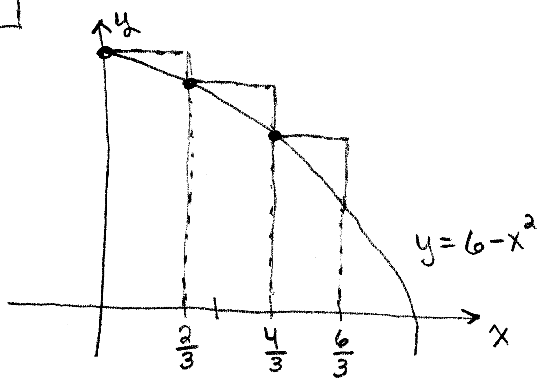


$$\sum_{k=1}^{2} f(c_k)\,\Delta x$$

$$= f(0)\,(1) + f(1)\,(1)$$

$$= 11$$

$\boxed{N = 3}$



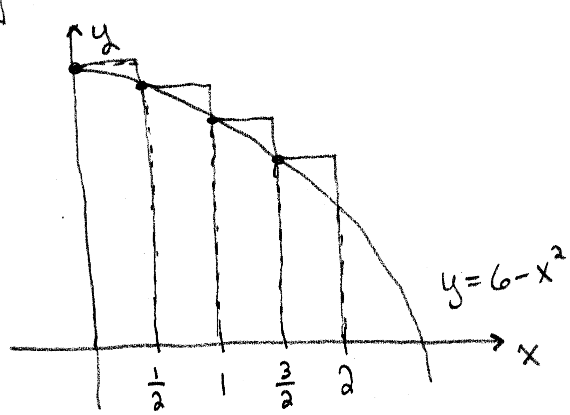
$\Delta x = \frac{2}{3}$

$$\sum_{k=1}^{3} f(c_k)\,\Delta x$$

$$= f(0)\left(\tfrac{2}{3}\right) + f\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right) + f\left(\tfrac{4}{3}\right)\left(\tfrac{2}{3}\right)$$

$$= \frac{284}{27} \approx 10.5185$$

$\boxed{N = 4}$



$\Delta x = \frac{1}{2}$

$$\sum_{k=1}^{4} f(c_k)\,\Delta x$$

$$= f(0)\left(\tfrac{1}{2}\right) + f\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) + f(1)\left(\tfrac{1}{2}\right)$$

$$+ f\left(\tfrac{3}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= \frac{41}{4} = 10.25$$

$$\left(\frac{2k-2}{N}\right)^2 = \frac{4k^2-8k+4}{N^2}$$

Arbitrary N

$$\Delta X = \frac{2-0}{N} = \frac{2}{N}$$

Partition: $0 < \frac{2}{N} < \frac{4}{N} < \frac{6}{N} < \cdots < \frac{2N}{N} = 2$

C_k = left endpoint of k^{th} subinterval

$$\Rightarrow \quad C_k = \frac{2k-2}{N}, \quad k = 1, 2, \ldots, N$$

$$\text{Riemann sum} = \sum_{k=1}^{N} f(c_k)\,\Delta x = \sum_{k=1}^{N} \left[6 - \left(\frac{2k-2}{N}\right)^2\right]\frac{2}{N}$$

$$= \sum_{k=1}^{N}\left(\frac{12}{N} - \frac{8k^2}{N^3} + \frac{16k}{N^3} - \frac{8}{N^3}\right)$$

$$= 12 - \frac{8}{N^3}\left[\frac{N(2N+1)(N+1)}{6}\right] + \frac{16}{N^3}\left[\frac{N(N+1)}{2}\right] - \frac{8}{N^2}$$

$$= 12 - \frac{4}{3}\left(\frac{2N^2+3N+1}{N^2}\right) + 8\left(\frac{N+1}{N^2}\right) - \frac{8}{N^2}$$

$$= 12 - \frac{8N^2-12N+4}{3N^2}$$

Exact Area

$$\lim_{N\to\infty}\left(12 - \frac{8N^2-12N+4}{3N^2}\right) = 12 - \frac{8}{3} = \frac{28}{3} = 9.\overline{3}$$