

Math 171 - Quiz 1

August 26, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. Use the quadratic formula to find the exact solutions of $x^2 - 4x + 1 = 0$.

$$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$x = 2 + \sqrt{3} \quad \text{or} \quad x = 2 - \sqrt{3}$$

2. Use your calculator to find the two smallest positive solutions of $x \sin x = 1$.

$$x \approx 1.114157, \quad x \approx 2.772605$$

3. Simplify. Then find all values of x for which $f(x) = 0$ or $f(x)$ DNE.

$f(x)$ DNE

$$f(x) = \frac{(12x+8)(x+1)^{3/2} - (9x^2+12x)(x+1)^{1/2}}{4(x+1)^3} = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$$

$f(x)$ DNE

$f(x) = 0$

$$(x+1)^{1/2} [(12x+8)(x+1) - (9x^2+12x)] = 0$$

WHEN THE DENOM

$f(x) = 0$

$$(x+1)^{1/2} (12x^2+20x+8-9x^2-12x) = 0$$

IS ZERO, WHICH

WHEN THE

$$(x+1)^{1/2} (3x^2+8x+8) = 0$$

IS AT $x = -1$.

NUMERATOR
IS ZERO...

↑
ZERO WHEN
 $x = -1$

↑ NEVER ZERO

$f(x)$ DNE
WHEN $x = -1$.

$f(x)$ IS NEVER
ZERO

4. Let $y = f(x) = x^3 - 2x + 1$. Find and simplify an expression for $\Delta y = f(x + \Delta x) - f(x)$.

$$\begin{aligned} \Delta y &= f(x + \Delta x) - f(x) = [(x + \Delta x)^3 - 2(x + \Delta x) + 1] - [x^3 - 2x + 1] \\ &= x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2x - 2\Delta x + 1 - x^3 + 2x - 1 \end{aligned}$$

$$= 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2\Delta x$$

5. Without using your graphing calculator, sketch the graphs of $f(x) = 4x - x^2$ and $g(x) = x^2 - 3x$. Then write and solve the equation that gives the x -coordinates of the two points of intersection.

$$f(x) = 4x - x^2 = x(4-x)$$

PARABOLA - opens DOWN -

X-INTERCEPTS AT $(0,0)$ & $(4,0)$ -

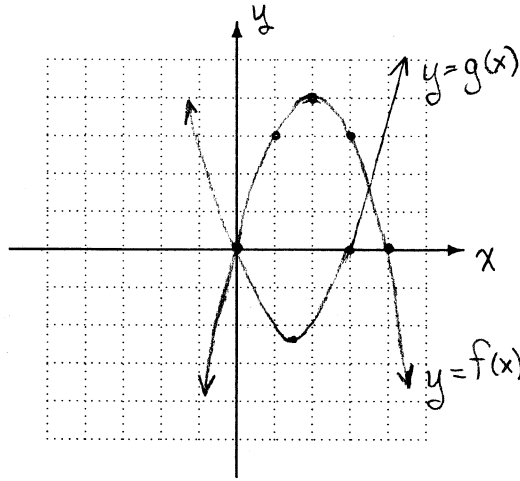
VERTEX AT $(2,4)$

$$g(x) = x^2 - 3x = x(x-3)$$

PARABOLA - opens UP -

X-INTERCEPTS AT $(0,0)$ & $(3,0)$

VERTEX AT $(1.5, -2.25)$



$$4x - x^2 = x^2 - 3x$$

$$2x^2 - 7x = 0$$

$$x(2x-7) = 0$$

$$x=0, x=\frac{7}{2}$$

6. Make sure your calculator is in radian mode. Construct a table showing the values of f at $x = \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$.

$$f(x) = \frac{3x^2}{\tan 4x^2}$$

What is a reasonable estimate for the limit at $x = 0$?

$f(x)$ IS AN EVEN FUNCTION

x	$f(x)$
± 0.1	0.7496
± 0.01	0.75
± 0.001	0.75
± 0.0001	0.75

$$\lim_{x \rightarrow 0} \frac{3x^2}{\tan 4x^2}$$

LOOKS LIKE

ABOUT 0.75.

7. Sketch the graph of a function f for which $f(1) = 2$ but $\lim_{x \rightarrow 1} f(x) = 3$.

