Math 171 - Quiz 3

September 9, 2010

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Consider the following piecewise function: $g(x) = \begin{cases} 3x^2 + ax, & x \leq 1 \\ 5 + b\cos\pi x, & x > 1 \end{cases}$

   (a) Find a so that $\lim_{x\to 1^-} g(x) = 6$.

   $$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} \left(3x^2 + ax\right) = 3 + a = 6 \Rightarrow \boxed{a = 3}$$

   (b) Find b so that $\lim_{x\to 4^+} g(x) = 13$.

   $$\lim_{x\to 4^+} g(x) = \lim_{x\to 4} \left(5 + b\cos\pi x\right) = 5 + b = 13 \Rightarrow \boxed{b = 8}$$

   (c) Find all possible a and b so that $\lim_{x\to 1} g(x)$ exists.

   MUST HAVE $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^-} g(x)$

   OR $5 + b\cos(\pi) = 3 + a$

   OR $5 - b = 3 + a$

   $\boxed{b = 2 - a,}$ $\boxed{a \text{ CAN BE ANY \#}}$

2. (4 points) Evaluate each limit.

   (a) $\lim_{x\to 2} \dfrac{x-2}{2-\sqrt{x+2}}$ $\frac{0}{0}$ $\cdot \dfrac{2+\sqrt{x+2}}{2+\sqrt{x+2}} = \lim_{x\to 2} \dfrac{(x-2)(2+\sqrt{x+2})}{4-(x+2)}$

   $$= \lim_{x\to 2} \frac{\cancel{(x-2)}(2+\sqrt{x+2})}{-1\,\cancel{(2-x)}} = -\left(2+\sqrt{4}\right) = \boxed{-4}$$

   (b) $\lim_{x\to 5^+} \dfrac{x^2-2x-15}{x^2-25}$

   $\frac{0}{0}$ $$\lim_{x\to 5^+} \frac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}(x+5)} = \frac{8}{10} = \boxed{\frac{4}{5}}$$

(c) $\lim_{x\to 0} \frac{7}{x \cot 5x} = \lim_{x\to 0} \frac{7 \sin 5x}{x \cos 5x} = \lim_{x\to 0} \frac{7}{\cos 5x} \cdot \lim_{x\to 0} \frac{\sin 5x}{x}$

$= \lim_{x\to 0} \frac{7}{\cos 5x} \cdot \lim_{x\to 0} \frac{5 \sin 5x}{5x} = (7)(5) = \boxed{35}$

3. (1 point) Suppose f is a function with the property that

$$-x^4 + 8x^3 - 24x^2 + 32x - 13 \le f(x) \le x^3 - 6x^2 + 12x - 5$$

for all x. What can be said about $\lim_{x\to 0} f(x)$?

Squeeze Thm $\Rightarrow$

Take $\lim_{x\to 0}$ all across.

(a) The limit does not exist.

(b) The limit is 3.

(c) If it exists, the limit is between -13 and -5.

(d) Nothing can be said about the limit.

$\Rightarrow -13 \le \lim_{x\to 0} f(x) \le -5$

4. (2 points) Sketch the graph (make it a nice graph!) of a function f such that

- $\lim_{x\to 2^+} f(x) = 3$
- $\lim_{x\to 2} f(x)$ exists
- $\lim_{x\to 4} f(x)$ DNE
- $\lim_{x\to 4^-} f(x) = -3$
- $f(2) = 0$
- $f(4) = 1$