Math 171 - Quiz 6

October 7, 2010

Name ___key___

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Differentiate: $\dfrac{d}{dt}\sqrt{\sin^3 5t}$

$$\frac{d}{dt}\left(\sin 5t\right)^{3/2} = \frac{3}{2}\left(\sin 5t\right)^{1/2}\left(\cos 5t\right)(5)$$

$$= \boxed{\frac{15}{2}\left(\cos 5t\right)\sqrt{\sin 5t}}$$

2. (4 points) Assume that y is implicitly defined as a function of x by the following equation. Find dy/dx at $(x, y) = (2, 1)$.

$$x^2 + y^4 - 2x^2 y = 1 - 2x$$

$$\frac{d}{dx}\left[x^2 + y^4 - 2x^2 y\right] = \frac{d}{dx}\left[1 - 2x\right]$$

$$2x + 4y^3\frac{dy}{dx} - 4xy - 2x^2\frac{dy}{dx} = -2$$

$$\left(4y^3 - 2x^2\right)\frac{dy}{dx} = 4xy - 2x - 2$$

$$\boxed{\frac{dy}{dx} = \frac{4xy - 2x - 2}{4y^3 - 2x^2}}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{8 - 4 - 2}{4 - 8} = \boxed{-\frac{1}{2}}$$

3. (3 points) The area of a circle is increasing at a rate of 3 ft^2/sec. What is the rate of change of circumference at the moment that the circumference is 24π ft?

A = AREA OF CIRCLE (FT^2)

t = TIME (SEC)

C = CIRCUMFERENCE (FT)

r = RADIUS (FT)

$\dfrac{dA}{dt} = 3$. FIND $\dfrac{dC}{dt}$ WHEN $C = 24\pi$

$A = \pi r^2$

$C = 2\pi r$

$\Downarrow$

$r = \dfrac{C}{2\pi}$

$A = \pi\left(\dfrac{C}{2\pi}\right)^2$

$A = \dfrac{C^2}{4\pi}$

$\dfrac{dA}{dt} = \dfrac{2C}{4\pi}\dfrac{dC}{dt}$

$3 = \dfrac{2(24\pi)}{4\pi}\dfrac{dC}{dt}$

$3 = 12\dfrac{dC}{dt}$

$\dfrac{dC}{dt} = \dfrac{3}{12} = \boxed{\dfrac{1}{4}\ \text{FT}/\text{SEC}}$