Math 171 - Quiz 8

October 28, 2010

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

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1. (6 points) Consider the function $f(x) = \dfrac{1}{x^2+1}$.
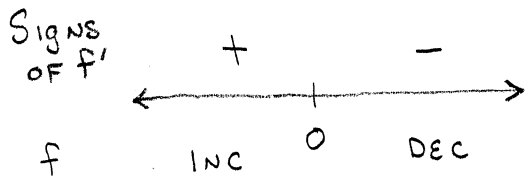
   f IS DEFINED FOR ALL X.

   - Find the critical numbers of f.
   - Find open intervals on which f is increasing/decreasing.
   - Identify all relative extreme values.
   - Find open intervals on which the graph of f is concave up/down.
   - Find all inflection points of f.
   - Sketch a detailed graph of f.

$$f'(x) = \frac{(x^2+1)(0) - (1)(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

$f'(x) = 0 \implies X = 0$

$f'(x)$ DNE NEVER

$\implies$ ONLY CRITICAL NUMBER IS $X = 0$

SIGNS OF f'

f INC 0 DEC

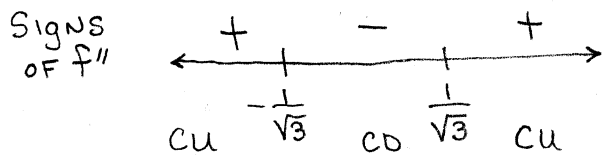f IS INC ON $(-\infty, 0)$

f IS DEC ON $(0, \infty)$

$f(0) = 1$ IS A RELATIVE MAX

$$f''(x) = \frac{(x^2+1)^2(-2) + 2x(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{-2(x^2+1) + 2x(2)(2x)}{(x^2+1)^3} = \frac{6x^2-2}{(x^2+1)^3}$$

GRAPH IS CD ON $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

GRAPH IS CU ON $\left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$

$f''(x) = 0 \implies 6x^2 = 2 \implies X = \pm\frac{1}{\sqrt{3}}$

$f''(x)$ DNE NEVER

INFLECTION POINTS

$\left(-\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$, $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

SIGNS OF f''

CU $-\frac{1}{\sqrt{3}}$ CD $\frac{1}{\sqrt{3}}$ CU

SEE ATTACHED SHEET FOR GRAPH.
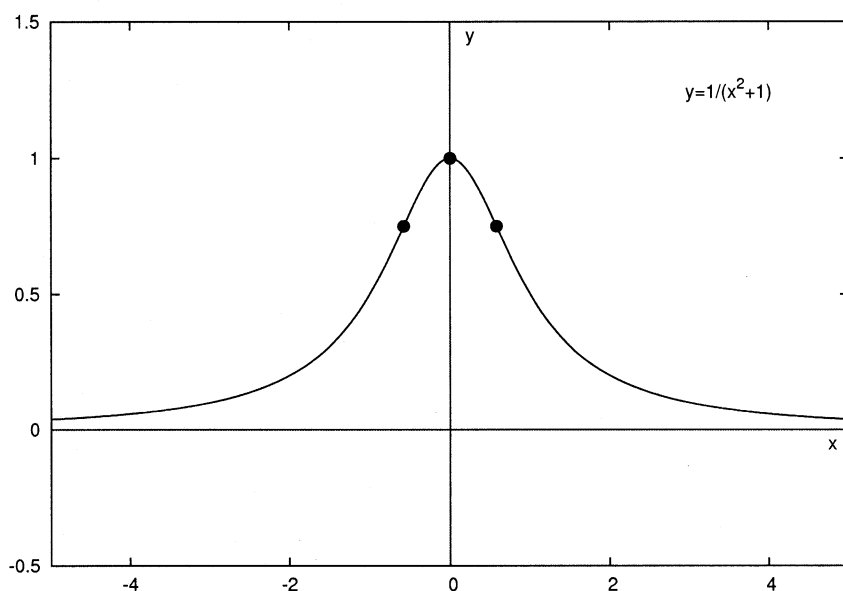
Graph of $f(x) = \dfrac{1}{x^2+1}$ ...

Since $f(-x) = f(x)$, the graph is
symmetric about the y-axis.

$f(0) = 1 \Rightarrow$ $(0,1)$ is the y-intercept.

$\displaystyle\lim_{x \to \pm\infty} \dfrac{1}{x^2+1} = 0 \Rightarrow$ $y = 0$ is a horizontal asymp.

$f(x) > 0 \Rightarrow$ graph is always above x-axis

$\Rightarrow$ No x-intercepts

All of this, along with the calculus analysis,
suggests the following graph.

2. (2 points) Give an example of a rational function whose graph has vertical asymptotes $x = -2$ and $x = 3$ and horizontal asymptote $y = 3/2$. Write your final answer as the quotient of two polynomials, with the numerator and denominator in expanded form.

$$R(x) = \frac{3x^2}{2(x+2)(x-3)} = \frac{3x^2}{2x^2 - 2x - 12}$$

3. (2 points) The table below gives the values of a function f and it first two derivatives at selected values of x. Determine which row gives the data for f, which row gives the data for f', and which row gives the data for f''. Explain your reasoning.

x	0.00	0.33	0.66	1.00	1.33	1.66	2.00	2.33	2.66	3.00
$A(x)$	0.00	0.64	1.14	1.38	1.28	0.84	0.08	−0.89	−1.91	−2.83
$B(x)$	0.00	0.11	0.41	0.84	1.30	1.66	1.82	1.69	1.22	0.42
$C(x)$	2.00	1.78	1.16	0.24	−0.83	−1.85	−2.65	−3.07	−3.00	−2.40

$f(x) = B(x)$

$f'(x) = A(x)$

$f''(x) = C(x)$

WHERE B IS INCREASING, A IS POSITIVE. WHERE B IS DECREASING, A IS NEGATIVE. THIS SUGGESTS

$$B'(x) = A(x)$$

WHERE A IS INCREASING, C IS POSITIVE. WHERE A IS DECREASING, C IS NEGATIVE. THIS SUGGESTS

$$A'(x) = C(x)$$

$$\left(\text{AND THEREFORE } B''(x) = C(x) \right)$$