

Math 171 - Quiz 9

November 11, 2010

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) On earth, the acceleration due to the force of gravity is given by $g = \frac{Gm}{r^2}$, where G is the gravitational constant, m is the mass of the earth, and r is the radius of the earth. Not worrying about the units, let's say G and m are measured exactly (no error):

$$G = 6.67428 \times 10^{-11} \quad \text{and} \quad m = 5.97370 \times 10^{24}.$$

Depending on how it is measured, the radius of the earth is between 6.357×10^6 and 6.378×10^6 . Use the average of these two numbers as your value for the radius. Compute g , and use differentials to approximate the error in your value.

$$r = 6.3675 \times 10^6, \quad \Delta r = 0.0105 \times 10^6$$

$$g = \frac{Gm}{r^2} \Rightarrow dg = -\frac{2Gm}{r^3} dr$$

↓

$$\Delta g \approx -\frac{2Gm}{r^3} \Delta r$$

$$g = \frac{(6.67428 \times 10^{-11})(5.97370 \times 10^{24})}{(6.3675 \times 10^6)^2} = 9.83354$$

$$\Delta g \approx -\frac{2Gm}{r^3} \Delta r = -0.03243$$

$$g = 9.83354 \pm 0.03243$$

2. (3 points) Evaluate each indefinite integral.

(a) $\int (3x^2 - 7x)^2 dx$

$$= \int (9x^4 - 42x^3 + 49x^2) dx$$

$$= \frac{9}{5}x^5 - \frac{42}{4}x^4 + \frac{49}{3}x^3 + C$$

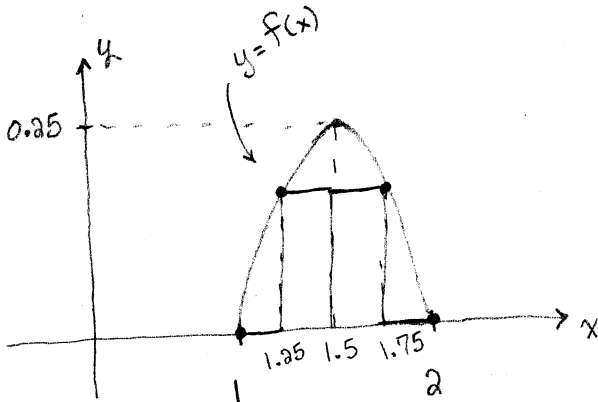
(b) $\int [4\sin(2x) + 7\cos(3x)] dx$

$$= -\frac{4}{2}\cos 2x + \frac{7}{3}\sin 3x + C$$

$$f(x) = -(x-1)(x-2)$$

3. (5 points) Let $f(x) = -x^2 + 3x - 2$. Your goal is to use 4 rectangles (of equal base length) to estimate the area of the region bounded between the graph of $y = f(x)$ and $y = 0$.

- (a) Under-estimate the area with rectangles underneath the curve. Draw the corresponding picture.

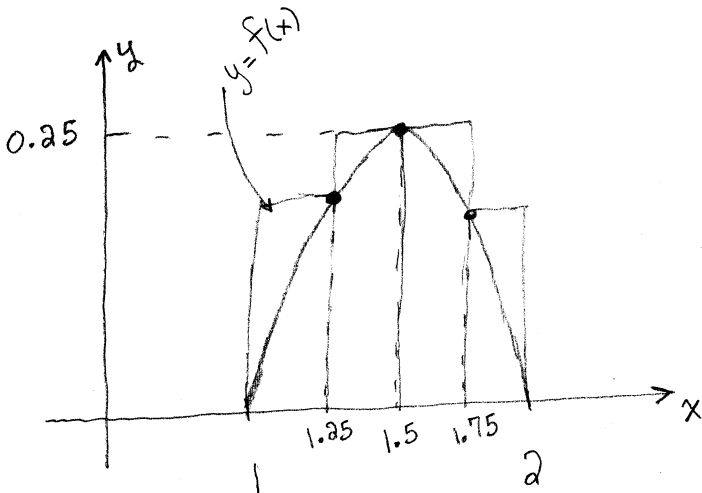


$$\Delta x = \frac{2-1}{4} = \frac{1}{4} = 0.25$$

UNDER ESTIMATE IS

$$\begin{aligned} &0.25 f(1) + 0.25 f(1.25) \\ &+ 0.25 f(1.75) + 0.25 f(2) \\ &= \boxed{0.09375} \end{aligned}$$

- (b) Over-estimate the area with rectangles above the curve. Draw the corresponding picture.



$$\Delta x = 0.25$$

OVER ESTIMATE IS

$$\begin{aligned} &0.25 f(1.25) + 0.25 f(1.5) \\ &+ 0.25 f(1.75) + 0.25 f(2) \\ &= \boxed{0.21875} \end{aligned}$$