# Math 171 - Test 1

September 23, 2010

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (4 points) Use a table of numerical values to estimate the following limit.

$$\lim_{x\to 0} \frac{3(1-\cos^2 x)}{5x}$$

$\frac{3(1-\cos^2 x)}{5x}$ IS AN ODD FUNCTION.

WE'LL CHECK +/_ SIMULTANEOUSLY.

LOOKS LIKE THE LIMIT IS 0.

| X | $\frac{3(1-\cos^2 x)}{5x}$ |
|---|---|
| $\pm 0.1$ | $\pm 0.0598$ |
| $\pm 0.01$ | $\pm 0.0059998$ |
| $\pm 0.001$ | $\pm 0.0006$ |
| $\pm 0.0001$ | $\pm 0.00006$ |

2. (10 points) Suppose that $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x)$ exists. Determine each limit.

(a) $\lim_{x\to 2} (x^2 f(x) + g(x)\sin \pi x)$

$$= 2^2 \left(\lim_{x\to 2} f(x)\right) + \left(\lim_{x\to 2} g(x)\right) \sin 2\pi$$

$$= 4\cdot 3 + (\text{Some } \#)(0) = \boxed{12}$$

(b) $\lim_{x\to 2} g(x)$ if $\lim_{x\to 2} \frac{1}{(g(x))^2} = 5$

RECIPROCAL $\Rightarrow \left(\lim_{x\to 2} g(x)\right)^2 = \frac{1}{5} \Rightarrow \lim_{x\to 2} g(x) = \boxed{\frac{1}{\sqrt{5}}}$

(c) $\lim_{x\to 2} g(x)$ if $\underbrace{\lim_{x\to 2} \frac{f(x)}{g(x)} \text{ does not exist}}$

ONLY WAY THIS IS POSSIBLE IS IF

$$\lim_{x\to 2} g(x) = \boxed{0}$$

3. (5 points) Each row of the table below gives some information about a function $f$. Fill in each blank entry with an appropriate word or number. In some cases there may be more than one correct answer.

| Continuous at $x = 2$ | $f(2)$ | $\lim_{x\to 2^-} f(x)$ | $\lim_{x\to 2^+} f(x)$ |
|---|---|---|---|
| Yes | 5 | 5 | 5 |
| No | 7 | Any # but 7 | 7 |
| No | Any # but -1 | -1 | -1 |
| Yes | 2 | 2 | 2 |
| Yes | 1 | 1 | 1 |

4. Consider the function $f(x) = 2x^2 + x$.

(a) (10 points) Use the limit definition of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^2 + (x+h)] - [2x^2 + x]}{h}$$

$$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + h}{h} = \lim_{h\to 0} (4x + 2h + 1)$$

$$= 4x + 1$$

(b) (5 points) Find an equation of the line tangent to the graph of $f$ at the point where $x = -2$.

$$x = -2 \Rightarrow y = f(-2) = 6$$

$$\text{Slope} = f'(-2) = -7$$

Line: $y - 6 = -7(x+2)$

or

$y = -7x - 8$

5. (24 points) Determine each limit or explain why the limit does not exist.

$\frac{0}{0}$ MORE WORK

(a) $\lim_{w\to -3} \frac{\sqrt{1-w}-2}{w+3} \cdot \frac{\sqrt{1-w}+2}{\sqrt{1-w}+2} = \lim_{w\to -3} \frac{1-w-4}{(w+3)(\sqrt{1-w}+2)}$

$= \lim_{w\to -3} \frac{-w-3}{(w+3)(\sqrt{1-w}+2)} = \lim_{w\to -3} \frac{-1}{\sqrt{1-w}+2} = \boxed{-\frac{1}{4}}$

(b) $\lim_{x\to 0} \left(\frac{7x^2 - \sin(4x)\cos(9x)}{4x}\right) = \lim_{x\to 0} \left(\frac{7x^2}{4x} - \frac{\sin 4x}{4x} \cos 9x\right)$

$= \lim_{x\to 0} \left(\frac{7x}{4}\right) - \left(\lim_{x\to 0} \frac{\sin 4x}{4x}\right)\left(\lim_{x\to 0} \cos 9x\right) = 0 - (1)(1)$

$= \boxed{-1}$

(c) $\lim_{t\to 6} \left(\frac{t-6}{t^2-12t+36}\right) = \lim_{t\to 6} \frac{1}{t-6}$

$\lim_{t\to 6^-} \frac{1}{t-6} = -\infty$ , $\lim_{t\to 6^+} \frac{1}{t-6} = +\infty$

$\frac{(+)}{(-)} = (-)$ $\quad$ $\frac{(+)}{(+)} = (+)$

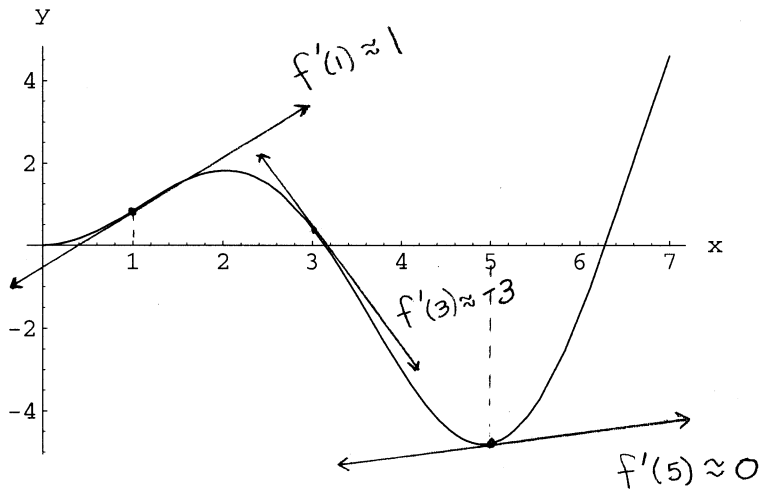
$\boxed{\lim_{t\to 6} \frac{1}{t-6} \text{ DNE}}$

(d) $\lim_{x\to \pi} (4x^2 - x\tan x)$

$= 4\pi^2 - \pi \tan \pi$ $\quad 0$ $= \boxed{4\pi^2}$

6. (5 points) The graph of the function $f$ is shown below. Referring to this graph, arrange the following quantities in ascending order.

$$\overset{\approx 1}{f'(1)},\quad \overset{\approx 0}{f'(5)},\quad \overset{\approx -5}{f(5)},\quad \overset{\approx 5}{f(7)},\quad \overset{\approx -3}{f'(3)}$$




Ascending order:
$f(5)$, $f'(3)$, $f'(5)$, $f'(1)$, $f(7)$

7. (4 points) Indicate whether each statement is true or false.

(a) ___T___ If $g$ is continuous at $x = 3$, then $\lim_{x \to 3^+} g(x) = g(3)$

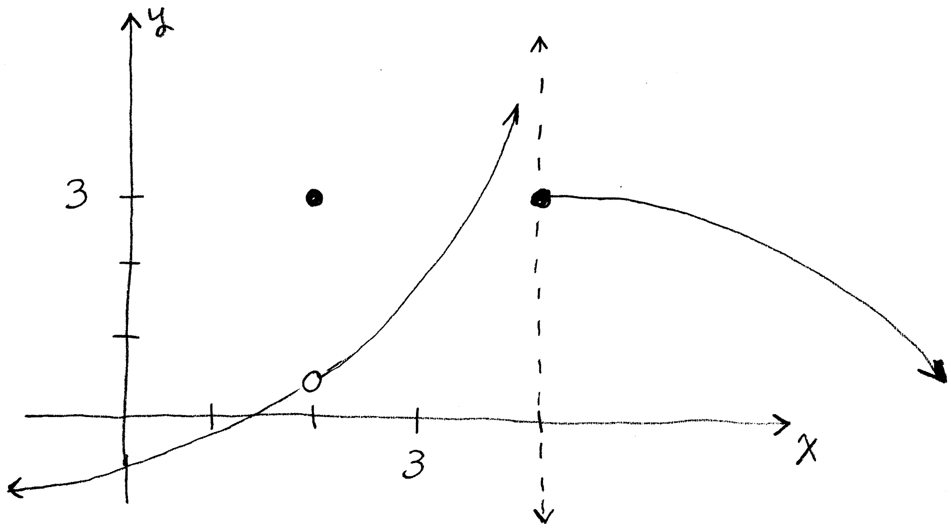
(b) ___F___ If $h$ has a limit at $x = -1$, then $h$ is defined at $x = -1$.

(c) ___F___ If $f$ and $g$ are polynomials and $g(6) = 0$, then the graph of $\frac{f(x)}{g(x)}$ must have a vertical asymptote at $x = 6$. eg $\frac{x-6}{x-6}$

(d) ___F___ If $g$ is defined at $x = 1$, then $g$ has a limit at $x = 1$.

8. (6 points) Sketch the graph of a function $f$ such that

- $f$ is defined for all real numbers,
- $f(2) = 3$,
- $f$ has a removable discontinuity at $x = 2$,
- $\lim_{x \to 4^-} f(x) = \infty$, and
- $\lim_{x \to 4^+} f(x) = 3$.




9. (10 points) Use basic differentiation rules (not the limit definition of derivative) to determine each derivative.

(a) $\frac{d}{dx}\left(9x^4 + 5 + \frac{2}{x^3}\right)$ $= \frac{d}{dx}\left(9x^4 + 5 + 2x^{-3}\right)$

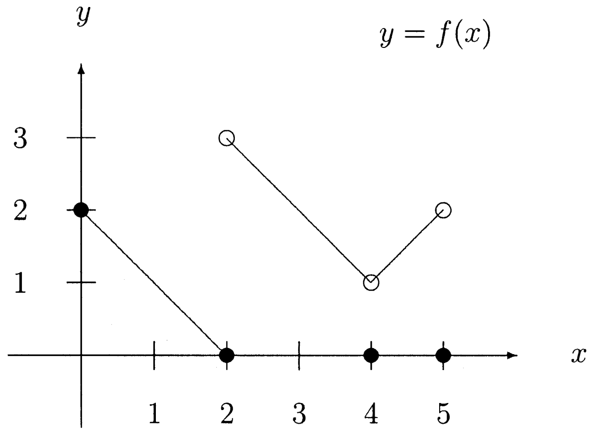
$= 36x^3 + 0 - 6x^{-4} = \boxed{36x^3 - \frac{6}{x^4}}$

(b) $\frac{d}{dx}\left(\sqrt[3]{x} + 6x - \sqrt{7}\cos x + \sin x\right)$

$= \frac{d}{dx}\left(x^{1/3} + 6x - \sqrt{7}\cos x + \sin x\right)$

$= \boxed{\frac{1}{3}x^{-2/3} + 6 + \sqrt{7}\sin x + \cos x}$

10. (7 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.




(a) $\lim_{x \to 4} f(x)$ = 1

(b) $f'(1)$ = −1 ( THE SLOPE AT X = 1 IS −1)

(c) $\lim_{x \to 2^-} f(x)$ = 0

(d) $f(5)$ = 0

(e) $\lim_{x \to 2^+} f(x)$ = 3

(f) The $x$-coordinate of a point at which $f'(x) > 0$

ANY NUMBER IN THE INTERVAL (4, 5)

(g) The $x$-coordinate of a point at which $f'(x) = 0$

NO SUCH POINTS - THE GRAPH OF f HAS NO HORIZONTAL TANGENT LINES.

11. (4 points) Using the definition of continuity as a guide, describe three reasons why a function may fail to be continuous at a point.

$f$ IS NOT CONTINUOUS AT $X = C$ IF

① $f(c)$ IS NOT DEFINED

② $\lim_{X \to C} f(x)$ DNE, OR

③ $\lim_{X \to C} f(x) \neq f(c)$

12. (6 points) You attempted to evaluate a limit by direct substitution, but you came up with 0/0. State three different ways that you may be able to resolve your indeterminate form. Then give an example illustrating any one of your ways.

① FACTOR AND CANCEL eg $\lim_{X \to 1} \frac{X^2 - 1}{X - 1} = \lim_{X \to 1} \frac{(X-1)(X+1)}{X-1}$

$$= \lim_{X \to 1} (X+1) = 2$$

② EXPAND AND SIMPLIFY

eg $\lim_{X \to 0} \frac{(1+X)^2 - 1}{X} = \lim_{X \to 0} \frac{1 + 2X + X^2 - 1}{X}$

$$= \lim_{X \to 0} \frac{2X + X^2}{X} = \lim_{X \to 0} (2+X)$$

$$= 2$$

③ USE $\lim_{X \to 0} \frac{\sin X}{X} = 1$

eg $\lim_{X \to 0} \frac{\sin 2X}{X} = \lim_{X \to 0} \frac{2 \sin 2X}{2X} = 2 \lim_{X \to 0} \frac{\sin 2X}{2X}$

$$= 2(1) = 2$$