Math 171 - Test 3a

November 18, 2010

Name key

Score

Show all work. Supply explanations when necessary.

1. (4 points) Find the horizontal asymptote of the graph of R.

$$R(x) = \frac{5x^3 - 8}{3x^3 + 2x^2 - 8x + 7}$$

$$\lim_{x \to \infty} \frac{5x^3 - 8}{3x^3 + 2x^2 - 8x + 7} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to \infty} \frac{5 - (8/x^3)}{3 + (2/x) - (8/x^2) + (7/x^3)}$$

$$= \frac{5}{3}$$

LIMIT AS $x \to -\infty$ IS SAME.

$\boxed{\text{HA IS } y = \frac{5}{3}.}$

2. (4 points) Find the function f such that $f'(x) = 3x^5 - 4x + 2$ and $f(1) = 3$.

$$f(x) = \int (3x^5 - 4x + 2)\,dx = \frac{3}{6}x^6 - \frac{4}{2}x^2 + 2x + C$$

$$f(1) = 3 \Rightarrow \frac{1}{2} - 2 + 2 + C = 3 \Rightarrow C = \frac{5}{2}$$

$$\boxed{f(x) = \frac{1}{2}x^6 - 2x^2 + 2x + \frac{5}{2}}$$

3. (3 points) Find the linearization of $g(x) = 3\sin 2x$ at $x = 0$.

$$L(x) = g(0) + g'(0)(x - 0)$$

$$g'(x) = 6\cos 2x$$

$$g(0) = 0$$

$$g'(0) = 6$$

$$\boxed{L(x) = 6x}$$

4. (4 points) Evaluate the limit: $\lim_{x\to-\infty} \frac{2x^2+7x-3}{5x^2-x^5}$

$$\lim_{x\to-\infty} \frac{2x^2+7x-3}{5x^2-x^5} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}} = \lim_{x\to-\infty} \frac{2/x^3 + 7/x^4 - 3/x^5}{5/x^3 - 1} = \frac{0}{-1}$$

$$= \boxed{0}$$

5. (4 points) What is the maximum number of horizontal asymptotes that a graph can have? Briefly explain your reasoning.

TWO ARE POSSIBLE, ONE IS OBTAINED BY FINDING A LIMIT AS $X \to +\infty$ AND ONE IS OBTAINED BY FINDING A LIMIT AS $X \to -\infty$.

6. (8 points) Evaluate each indefinite integral.

(a) $\int \left(\sqrt[5]{x^3} - 5x^2 + \frac{4}{x^3}\right) dx = \int \left(x^{3/5} - 5x^2 + 4x^{-3}\right) dx$

$$= \boxed{\frac{5}{8}x^{8/5} - \frac{5}{3}x^3 - 2x^{-2} + C}$$

(b) $\int (3\sec^2 t - \cos 7t)\, dt$

$$= \boxed{3\tan t - \frac{1}{7}\sin 7t + C}$$

7. (12 points) A rectangle is inscribed in the first quadrant region below the graph of $y = 6 - x^3$ (see below). Find the coordinates of the point (x, y) that maximize the area of the rectangle.

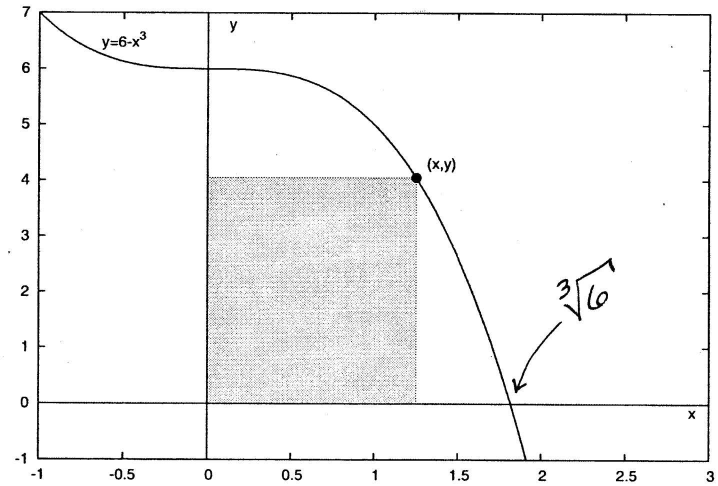
LET

X = X-COORD OF PT

= BASE OF RECT

y = y-COORD OF PT

= HEIGHT OF RECT



$\sqrt[3]{6}$

MAXIMIZE AREA

$A = xy$

SUBJECT TO $y = 6 - x^3$.

$A(x) = x(6 - x^3)$

$= 6x - x^4, \quad 0 \leq x \leq \sqrt[3]{6}$

≈ 1.817

$A'(x) = 6 - 4x^3 = 0$

$\Rightarrow x^3 = \frac{6}{4} \Rightarrow x = \sqrt[3]{\frac{3}{2}}$

≈ 1.145

SIGNS OF A'

$+ \qquad -$

$0 \qquad \sqrt[3]{\frac{3}{2}} \qquad \sqrt[3]{6}$

GIVES A MAX $\Rightarrow$

POINT IS $\left(\sqrt[3]{\frac{3}{2}}, \frac{9}{2}\right)$

8. (4 points) Some values of $f(x)$ and $f'(x)$ near $x = 1$ are given in the table below. Find the linearization of f at $x = 1$. Then use the linearization to approximate $f(0.75)$.

x	0.50	0.75	1.00	1.25	1.50
$f(x)$	6.08	6.90	8.00	9.41	11.14
$f'(x)$	2.74	3.82	5.00	6.26	7.60

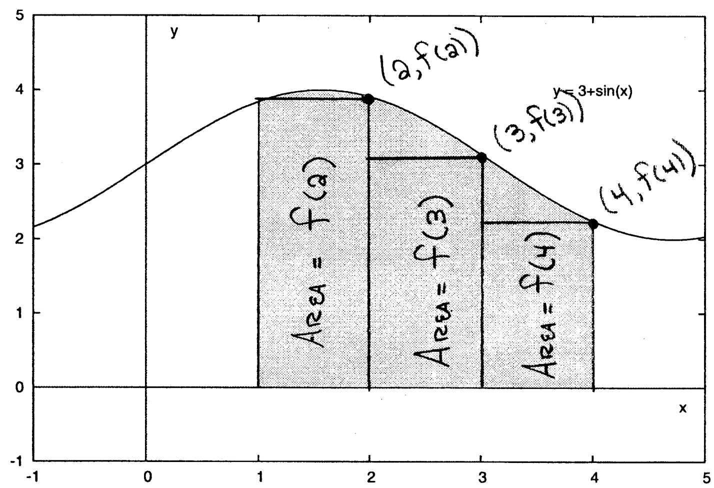
$L(x) = f(1) + f'(1)(x-1) = 8 + 5(x-1) = 8 + 5x - 5$

$\boxed{L(x) = 5x + 3}$

$f(0.75) \approx L(0.75) = 5(0.75) + 3$

$= 6.75$

9. (10 points) The graph of $f(x) = 3 + \sin x$ is shown below.



(a) Write the definite integral that gives the area of the shaded region.

$$\int_1^4 (3 + \sin x)\, dx$$

(b) Use three subintervals of equal length and right subinterval endpoints to compute a Riemann sum that approximates the value of your definite integral.

$\Delta x = \frac{4-1}{3} = 1$, RIGHT ENDPOINTS $\Rightarrow$ $c_1 = 2$, $c_2 = 3$, $c_3 = 4$

RIEMANN SUM $= f(2)(1) + f(3)(1) + f(4)(1)$

≈ 9.2936

(c) Sketch the rectangles associated with your Riemann sum on the graph above. Does your Riemann sum appear to under-estimate or over-estimate the exact value of the definite integral.

SEE ABOVE. THE RIEMANN SUM UNDER-ESTIMATES THE EXACT VALUE.

10. (5 points) Use Newton's method, starting with $x_0 = -1$, to approximate the solution of the equation $x = \cos x$. Which one of these numbers is closest to your value of x_2?

(a) 0.75
(b) 2.98
(c) 8.72
(d) −0.51

$f(x) = x - \cos x$

$f'(x) = 1 + \sin x$

$$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$$

$x_0 = -1$

$x_1 = 8.716217$

$x_2 = 2.9760655$

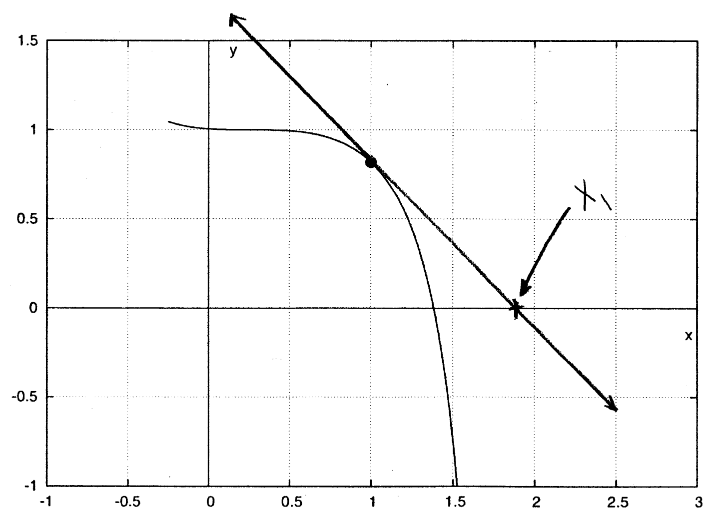
11. (4 points) Let $V = 6\cos(3w^2 + 5)$. Determine the differential dV.

$$dV = 6\left(-\sin(3w^2+5)(6w)\right)\,dw$$

$$\boxed{dV = -36w\sin(3w^2+5)\,dw}$$

12. (3 points) The graph of $y = f(x)$ is shown below. Suppose you use Newton's method, starting with $x_0 = 1$, to approximate a solution of $f(x) = 0$. Which one of the following numbers would be closest to x_1?

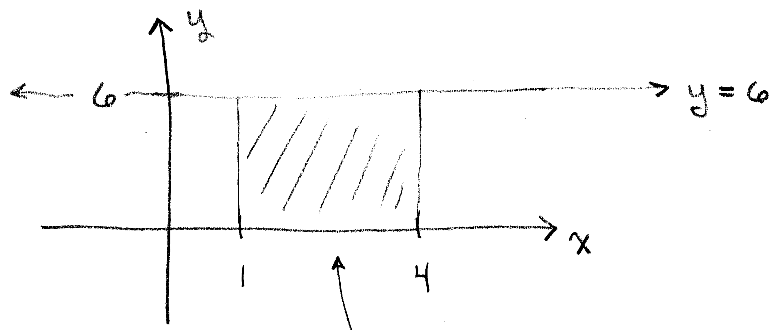
(a) 1.9
(b) 1.35
(c) 2.5
(d) 1.0



13. (10 points) Use area to evaluate each definite integral.

(a) $\int_1^4 6\,dx$

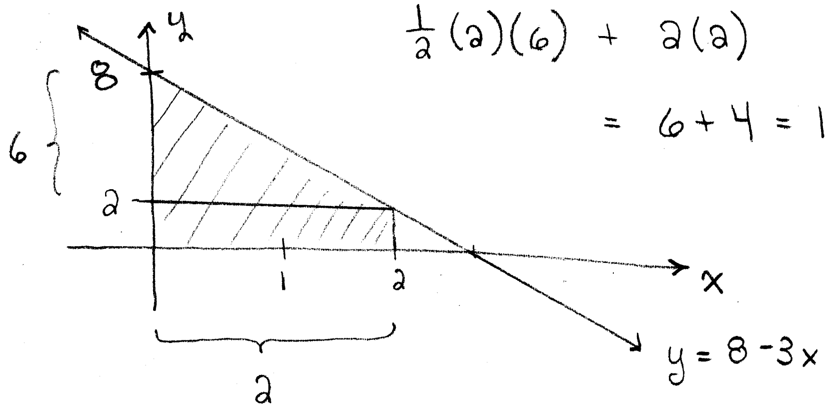
$$\boxed{\int_1^4 6\,dx = 18}$$



Area $= 3(6) = 18$

(b) $\int_0^2 (8-3x)\,dx$

$$\boxed{\int_0^2 (8-3x)\,dx = 10}$$



$\frac{1}{2}(2)(6) + 2(2)$

$= 6 + 4 = 10$

14. (5 points) Evaluate the limit: $\lim_{x\to\infty} \dfrac{3x}{\sqrt{2x^2+5}}$.

$$\lim_{x\to\infty} \frac{3x}{\sqrt{2x^2+5}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{3x}{\sqrt{2x^2+5}} \cdot \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{2+\frac{5}{x^2}}} = \boxed{\frac{3}{\sqrt{2}}}$$

Math 171 - Test 3b

November 18, 2010

Name ~~~~key~~~~   Score ______

Show all work. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

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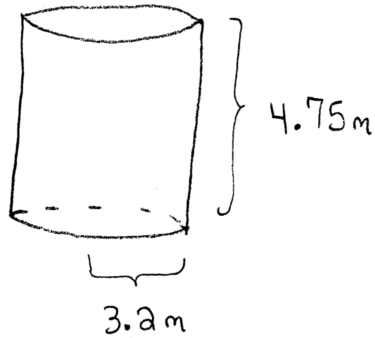
1. (4 points) A right circular cylinder has height 4.75 m. Use differentials to estimate the change in volume if the radius changes from 3.2 m to 3.3 m. What is the corresponding percent change?



$V = 4.75\pi r^2$

$dV = 9.5\pi r\, dr$

$\Delta V \approx 9.5\pi r\, \Delta r$

r CHANGES FROM $\underbrace{3.2 \text{ TO } 3.3}_{\Delta r = 0.1}$

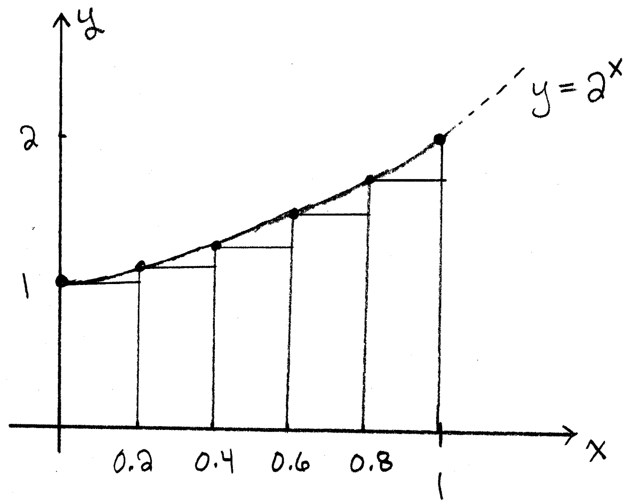
$\Delta V \approx 9.5\pi (3.2)(0.1)$

$\approx \boxed{9.55 \text{ m}^3}$

% CHANGE $= \dfrac{9.55}{4.75\pi (3.2)^2} \times 100\%$

$\approx \boxed{6.25\%}$

2. (6 points) Let $f(x) = 2^x$. Using 5 subintervals of equal length, compute a Riemann sum for $f(x)$ on $[0, 1]$. Sketch the graph along with your rectangles.



$\Delta x = \dfrac{1-0}{5} = \dfrac{1}{5} = 0.2$

I'LL USE LEFT ENDPOINTS...

$c_1 = 0,\ c_2 = 0.2,\ c_3 = 0.4,$

$c_4 = 0.6,\ c_5 = 0.8$

RIEMANN SUM =

$0.2\big(f(0) + f(0.2) + f(0.4)$

$+ f(0.6) + f(0.8)\big)$

≈ 1.345

3. (10 points) Use algebra and calculus techniques to sketch the graph of $y = \frac{x^3}{3} - 3x$. Use graph paper! (This problem is open-ended on purpose. Be thorough enough to make your work worth 10 points. Graph paper is available at `http://www.printfreegraphpaper.com`.)

$$f(x) = \frac{x^3}{3} - 3x = \frac{1}{3}x(x^2-9) = \frac{1}{3}x(x-3)(x+3)$$

X-INTERCEPTS: $(0,0)$, $(3,0)$, $(-3,0)$

Y-INTERCEPT: $(0,0)$

CUBIC POLY WITH POS. LEADING COEFF: $\lim_{x\to\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

NO HORIZONTAL, VERTICAL, OR SLANT ASYMPTOTES.

$$f(-x) = \frac{(-x)^3}{3} - 3(-x) = -\frac{x^3}{3} + 3x = -f(x) \Rightarrow$$ f IS ODD. GRAPH IS SYMMETRIC ABOUT ORIGIN.

$$f'(x) = x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$$

SIGNS OF f': $+$ on $(-\infty, -\sqrt{3})$, $-$ on $(-\sqrt{3}, +\sqrt{3})$, $+$ on $(+\sqrt{3}, \infty)$

f IS INCREASING ON $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

f IS DECREASING ON $(-\sqrt{3}, \sqrt{3})$

$f(-\sqrt{3}) = 2\sqrt{3} \approx 3.5$ IS A REL MAX

$f(\sqrt{3}) = -2\sqrt{3} \approx -3.5$ IS A REL MIN

$$f''(x) = 2x = 0 \Rightarrow x = 0$$

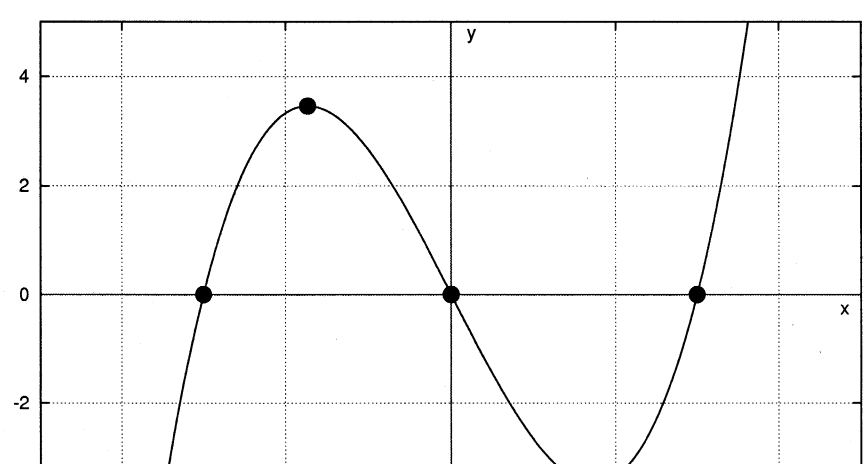
SIGNS OF f'': $-$ on $(-\infty, 0)$, $+$ on $(0, \infty)$

GRAPH IS CD ON $(-\infty, 0)$

GRAPH IS CU ON $(0, \infty)$

$(0,0)$ IS AN INFLECTION POINT.

SEE ATTACHED GRAPH.



y
4
2
0
-2
-4
x
-4
-2
0
2
4