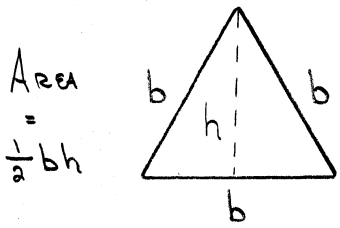
**Math 171 - Quiz 10**
November 8, 2012

Name key

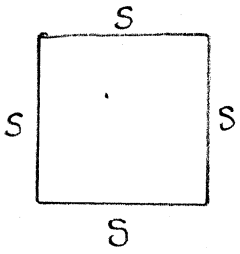
Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Problem #42a, page 225




Area = $\frac{1}{2}bh$




Area = $s^2$

$h = \sqrt{b^2 - \frac{b^2}{4}}$

Maximize $A = \frac{1}{2}b\sqrt{b^2 - \frac{b^2}{4}} + s^2$

s.t. $3b + 4s = 20$

$A = \frac{1}{2}b\sqrt{\frac{3b^2}{4}} + s^2 = \frac{\sqrt{3}\,b^2}{4} + s^2$

$3b + 4s = 20 \Rightarrow s = \frac{20 - 3b}{4}$

$A(b) = \frac{\sqrt{3}\,b^2}{4} + \left(\frac{20 - 3b}{4}\right)^2$

Domain of A: $0 \le b \le \frac{20}{3}$

Maximize $A(b) = \frac{\sqrt{3}\,b^2}{4} + \left(\frac{20 - 3b}{4}\right)^2$ on $[0, \frac{20}{3}]$

$A'(b) = \frac{\sqrt{3}}{2}b + 2\left(\frac{20 - 3b}{4}\right)\left(-\frac{3}{4}\right)$

$= \frac{\sqrt{3}}{2}b + \frac{1}{8}(9b - 60)$

$= \left(\frac{\sqrt{3}}{2} + \frac{9}{8}\right)b - \frac{60}{8} = 0$

$\Rightarrow b = \frac{60/8}{\frac{\sqrt{3}}{2} + 9/8} = \frac{60}{4\sqrt{3} + 9} \approx 3.767$

$A(0) = 25 \leftarrow$ Abs max

$A\left(\frac{20}{3}\right) \approx 19.245$

$A\left(\frac{60}{4\sqrt{3}+9}\right) \approx 10.874$

For a max, no wire should go to the triangle. All should go to the square to make a 5' x 5' square.

2. (5 points) Problem #20, page 215

$g(x) = x\sqrt{9-x^2}$

- DOMAIN: $-3 \le x \le 3$
- INTERCEPTS: $g(x) = 0 \Rightarrow x = 0,\ x = 3,\ x = -3$

X-INTERCEPTS ARE $(0,0)$, $(3,0)$, $(-3,0)$

Y-INTERCEPT IS $(0,0)$.

- THE GRAPH HAS NO ASYMPTOTES.
- SINCE $g(-x) = (-x)\sqrt{9-(-x)^2}$
$= -x\sqrt{9-x^2} = -g(x)$

THE GRAPH IS SYMMETRIC ABOUT THE ORIGIN.

- FIRST DERIVATIVE...

$g'(x) = x\left(\frac{1}{2}\right)(9-x^2)^{-1/2}(-2x) + (9-x^2)^{1/2}$

$= (9-x^2)^{-1/2}\left(-x^2 + 9 - x^2\right)$

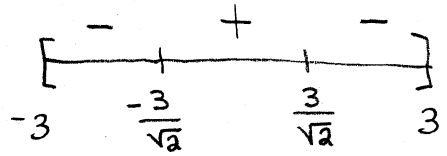
$g'(x) = (9-2x^2)(9-x^2)^{-1/2}$

$g'(x)$ DNE WHEN $x = 3$ OR $x = -3$

$g'(x) = 0 \Rightarrow 9 = 2x^2 \Rightarrow$

$x = \frac{3}{\sqrt{2}}$ OR $x = -\frac{3}{\sqrt{2}}$

SIGNS OF $g'$




$g$ IS INCREASING ON $\left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

$g$ IS DECREASING ON $\left(-3, -\frac{3}{\sqrt{2}}\right) \cup \left(\frac{3}{\sqrt{2}}, 3\right)$

$g\left(-\frac{3}{\sqrt{2}}\right) = -4.5$

IS A REL MIN / ABS MIN

$g\left(\frac{3}{\sqrt{2}}\right) = 4.5$

IS A REL MAX / ABS MAX

- SECOND DERIVATIVE...

$g''(x) = -4x(9-x^2)^{-1/2} + (9-2x^2)\left(-\frac{1}{2}\right)(9-x^2)^{-3/2}(-2x)$

$= (9-x^2)^{-3/2}\left[-4x(9-x^2) + x(9-2x^2)\right]$

$g''(x) = (9-x^2)^{-3/2}\left(-27x + 2x^3\right)$

$g''(x)$ DNE WHEN $x = 3$ OR $x = -3$

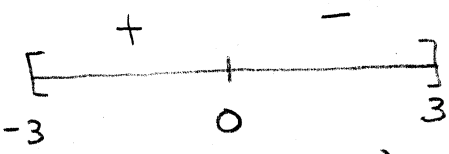
$g''(x) = 0 \Rightarrow 2x^3 - 27x = 0$

$x(2x^2 - 27) = 0$

$x = 0,\ x = \sqrt{\frac{27}{2}},\ x = -\sqrt{\frac{27}{2}}$

OUTSIDE DOMAIN OF $g$.

SIGNS OF $g''$

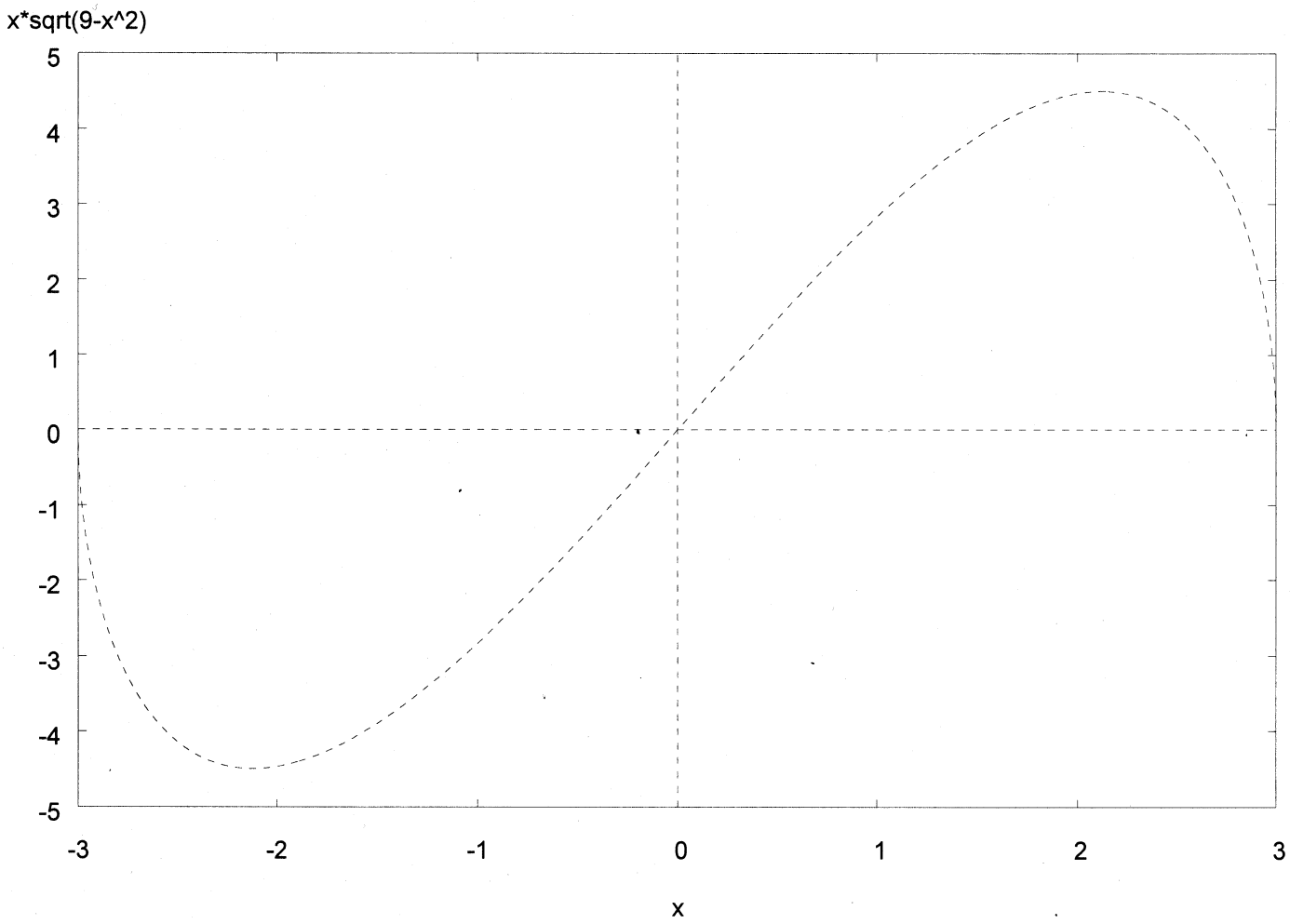



GRAPH IS CU ON $(-3,0)$

GRAPH IS CD ON $(0,3)$

$(0,0)$ IS THE ONLY INFLECTION POINT.

SEE ATTACHED GRAPH.


x*sqrt(9-x^2)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-3
-2
-1
0
1
2
3
x