Math 171 - Quiz 4

September 13, 2012

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Consider the following piecewise function: $g(x) = \begin{cases} 5x^2 - ax, & x \le 2 \\ 15 + b\cos(\pi x), & x > 2 \end{cases}$

   (a) Find a so that $\lim\limits_{x \to 2^-} g(x) = 6$.

   $$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (5x^2 - ax) = 20 - 2a = 6 \implies \boxed{a = 7}$$

   (b) Find a and b so that $g(2) = 12$ and g is continuous at $x = 2$.

   $$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x) = 2 \implies 20 - 2a = 15 + b = 12$$

   $$\boxed{a = 4 \text{ AND } b = -3}$$

2. (4 points) Evaluate each limit.

   (a) $\lim\limits_{x \to -4^+} \dfrac{x-2}{\sqrt{12-x}} = \dfrac{-4-2}{\sqrt{12-(-4)}} = \dfrac{-6}{4} = \boxed{-\dfrac{3}{2}}$

   (b) $\lim\limits_{x \to 5^-} \dfrac{x^2 - 2x - 15}{x^2 - 25} \overset{\frac{0}{0}}{=} \lim\limits_{x \to 5^-} \dfrac{(x-5)(x+3)}{(x-5)(x+5)} = \dfrac{8}{10} = \boxed{\dfrac{4}{5}}$

3. (2 points) Find and classify the discontinuities of $f(x) = \dfrac{\sin x}{x(x+5)^2}$.

   ONLY DISCONTINUITIES ARE WHERE $f(x)$ DNE: $X = 0$, $X = -5$.

   DISCONT AT $X = 0$ IS REMOVEABLE

   BECAUSE

   $\lim\limits_{x \to 0} \dfrac{\sin x}{x} \cdot \dfrac{1}{(x+5)^2} = (1)\left(\dfrac{1}{25}\right)$

   DISCONT AT $X = -5$ IS INFINITE, NONREMOVEABLE BECAUSE

   $\lim\limits_{x \to -5} \dfrac{\sin x}{x(x+5)^2} = -\infty$