Math 171 - Quiz 7
October 11, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find an equation of the line tangent to the graph of $y = x\sqrt{x^3+1}$ at the point where $x = 2$.

$y = x(x^3+1)^{1/2}$

$$\frac{dy}{dx} = x\left(\tfrac{1}{2}\right)(x^3+1)^{-1/2}(3x^2) + (x^3+1)^{1/2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=2} = 2\left(\tfrac{1}{2}\right)(9)^{-1/2}(12) + (9)^{1/2}$$

$$= 4 + 3 = 7$$

$y - 6 = 7(x-2)$

POINT: $x = 2 \Rightarrow y = 2(9)^{1/2} = 6$

2. (3 points) The graph of $y^2 - xy^2 - x^3 = 0$ is called a Cissoid of Diocles. Find dy/dx.

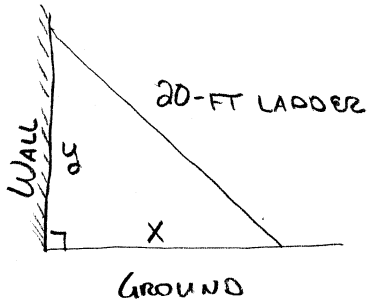
$$\frac{d}{dx}(y^2 - xy^2 - x^3) = 0$$

$$2y\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} - 3x^2 = 0$$

$$2y\frac{dy}{dx} - 2xy\frac{dy}{dx} = 3x^2 + y^2$$

$$\frac{dy}{dx} = \frac{3x^2+y^2}{2y-2xy}$$

3. (4 points) A 20-ft ladder leaning against a wall begins to slide. How fast is the top of the ladder sliding down the wall at the moment when the bottom of the ladder is 12 ft from the wall and sliding away from the wall at a rate of 5 ft/sec?



y = HEIGHT OF TOP OF LADDER AT TIME t

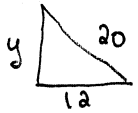
x = DISTANCE BASE IS FROM WALL AT TIME t

$\frac{dx}{dt} = 5$. FIND $\frac{dy}{dt}$ WHEN $x = 12$

$$x^2 + y^2 = 20^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y} = -\frac{x}{y}\frac{dx}{dt}$$

WHEN $x = 12$,



$y^2 = 400 - 144$

$y = \sqrt{256}$

$$\frac{dy}{dt} = -\frac{12}{\sqrt{256}} \cdot 5 \approx -3.75 \text{ FT/SEC}$$