Math 171 - Test 1
September 20, 2012

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

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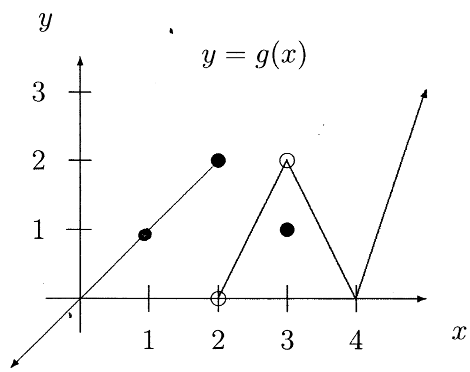
1. (7 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x\to 1} g(x)$ = 1

(b) $\lim_{x\to 2^+} g(x)$ = 0

(c) $\lim_{x\to 2} g(x)$ DNE (LIMIT FROM LEFT ≠ LIMIT FROM RIGHT)

(d) $g'(0)$ = 1 (THE RAY THROUGH (0,0) HAS SLOPE 1)

(e) $g(3)$ = 1

(f) $\lim_{x\to 3} g(x)$ = 2

(g) $\lim_{x\to 0^-} g(x)$ = 0

2. (9 points) Suppose that $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x)$ exists. Determine each limit.

(a) $\lim_{x\to 1} [xf(x) + \sin \pi x]$

$$= (1)\left[\lim_{x\to 1} f(x)\right] + (\sin \pi)$$

$$= (1)(4) + 0 = \boxed{4}$$

(b) $\lim_{x\to 1} g(x)$ if $\lim_{x\to 1} [g(x)\ \cos \pi x] = 8$

$$\left[\lim_{x\to 1} g(x)\right](\cos \pi) = 8 \Rightarrow \lim_{x\to 1} g(x) = \frac{8}{\cos \pi} = \boxed{-8}$$

(c) $\lim_{x\to 1} g(x)$ if $\underbrace{\lim_{x\to 1} \frac{f(x)}{g(x)} \text{ does not exist}}$

THIS MUST MEAN $\lim_{x\to 1} g(x) = \boxed{0}$

3. (6 points) Consider the rational function $R(x) = \dfrac{3x+3}{x^2+3x+2}$. Find all points at which R is discontinuous, and state whether each discontinuity is removable or nonremovable.

$$R(x) = \frac{3(x+1)}{(x+1)(x+2)}$$

DISCONTINUITIES AT $X = -1$ AND $X = -2$

DISCONT AT $X=-1$ IS REMOVABLE BECAUSE $\lim_{X\to -1} R(x) = 3$

DISCONT AT $X=-2$ IS NONREMOVABLE BECAUSE $\lim_{X\to -2} R(x)$ DNE.

4. (4 points) Indicate whether each statement is true or false.

(a) F If f has a limit at $x = 2$, then f is defined at $x = 2$.

(b) T If g is continuous at $x = -1$ then $\lim_{x \to -1^+} g(x) = g(-1)$

(c) T If $f(x) \geq 0$ for all x and $\lim_{x \to 0} f(x) = 0$, then $\lim_{x \to 0} \frac{1}{f(x)} = \infty$

(d) F If f is defined at $x = 2$, then f has a limit at $x = 2$.

5. (24 points) Determine each limit or explain why the limit does not exist.

(a) $\lim_{x \to 0} \frac{5x - 4\sin 3x}{2x} = \lim_{x \to 0} \frac{5x}{2x} - \frac{4 \cdot 3}{2} \lim_{x \to 0} \frac{\sin 3x}{3x} = \frac{5}{2} - 6(1)$
$= \boxed{-\frac{7}{2}}$

(b) $\lim_{k \to 4} \frac{\sqrt{k} - 2}{k - 4} \cdot \frac{\sqrt{k} + 2}{\sqrt{k} + 2} = \lim_{k \to 4} \frac{\cancel{k-4}}{(\cancel{k-4})(\sqrt{k} + 2)} = \boxed{\frac{1}{4}}$

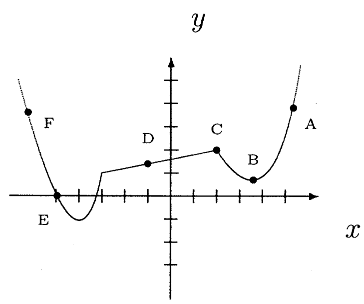
(c) $\lim_{r \to 5} \left(\frac{r - 5}{r^2 - 10r + 25} \right) = \lim_{r \to 5} \frac{r-5}{(r-5)^2} = \lim_{r \to 5} \frac{1}{r-5}$ $\frac{k \neq 0}{0}$ DNE BECAUSE

$\lim_{r \to 5^+} \frac{1}{r-5} \overset{+}{\underset{+}{=}} +\infty \neq \lim_{r \to 5^-} \frac{1}{r-5} \overset{+}{\underset{-}{=}} -\infty$

(d) $\lim_{w \to 6^+} \frac{w^2 - 40}{6 - w}$ $\frac{-4}{0}$, $\frac{-}{-}$

LIMIT IS $+\infty$.

6. (6 points) Consider the function f whose graph is shown below.



Referring to the labeled points, find a point at which

(a) $f'(x) = 0$

B

(b) $0 < f'(x) < 1$

D

(c) $f'(x) > 1$

A

(d) $f(x) = 0$

E

(e) $f'(x) < 0$

E OR F

(f) $f'(x)$ is not defined

C

7. (12 points) Let $f(x) = x^2 - 2x$. Use the limit definition of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left[(x+h)^2 - 2(x+h)\right] - \left[x^2 - 2x\right]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h} = \lim_{h \to 0} \frac{2xh + h^2 - 2h}{h}$$

$$= \lim_{h \to 0} (2x + h - 2) = \boxed{2x - 2}$$

8. (4 points) Give an example of a function whose graph has vertical asymptotes at $x = 1$ and $x = 5$.

$$f(x) = \frac{1}{(x-1)(x-5)}$$

9. (4 points) What does it mean for a discontinuity to be removable?

THE DISCONTINUITY AT $X = C$ IS REMOVABLE IF $\lim_{x \to c} f(x)$ EXISTS.

10. (10 points) Let $f(x) = 2x^3 - x^2$. Use basic differentiation rules (not the definition) to find $f'(x)$. Then find an equation of the line tangent to the graph of f at $x = 2$.

$$f'(x) = 2(3)x^2 - 2x^1 = \boxed{6x^2 - 2x}$$

$$m = f'(2) = 6(4) - 4 = 20$$

POINT: $x = 2$

$$y = f(2) = 2(8) - 4 = 12$$

TANGENT LINE:

$$\boxed{y - 12 = 20(x - 2)}$$

11. (8 points) The table belows gives the values of the **continuous** function f at selected points.

x	-2	-1	0	1	2
$f(x)$	1.5	2.75	2.1	-1.75	-3.4

(a) Find $\lim_{x\to 0} f(x)$. Explain your reasoning.

SINCE f IS CONTINUOUS,

$$\lim_{x\to 0} f(x) = f(0) = 2.1$$

(b) Find an interval on which you can be sure the equation $f(x) = 1$ has at least one solution. Explain your reasoning.

INTERMEDIATE VALUE THEOREM!

SINCE f IS CONTINUOUS AND

$f(0) > 1$ AND $f(1) < 1$,

$f(x)$ MUST EQUAL ONE SOMEWHERE BETWEEN 0 AND 1.

12. (6 points) Use a table of numerical values to estimate the limit. Your table should show at function values at six or more points.

$$\lim_{z\to 0} \frac{2^z - 1}{z}$$

z	$\frac{2^z-1}{z}$
0.1	0.71773
0.01	0.69556
0.001	0.69339
-0.1	0.66967
-0.01	0.69075
-0.001	0.69291

IT LOOKS LIKE

$$\lim_{z\to 0} \frac{2^z-1}{z} \approx 0.693$$