Math 171 - Test 3a
November 15, 2012

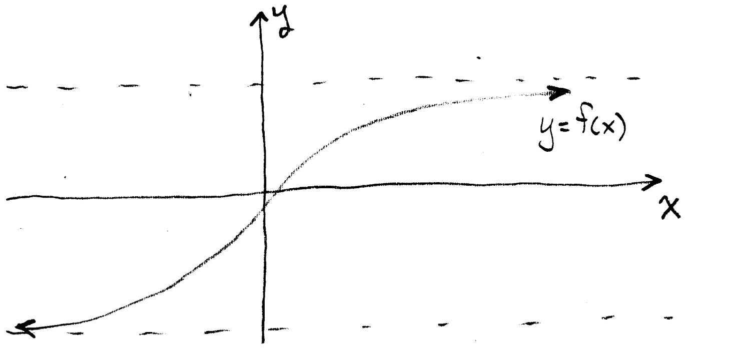
Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) What is the maximum number of horizontal asymptotes that the graph of a function can have? Sketch the graph of a function with that number of horizontal asymptotes.

MAX NUMBER OF HORIZONTAL ASYMPTOTES IS 2.



GRAPH OF f HAS TWO HORIZONTAL ASYMPTOTES.

2. (6 points) Let $f(x) = x^3 + \sin(10x)$. Without looking at the graph of f, determine whether the graph is concave up or down at the point where $x = 0.65$.

$$f'(x) = 3x^2 + 10\cos(10x)$$

$$f''(x) = 6x - 100\sin(10x)$$

$$f''(0.65) = -17.6119988I < 0$$

$\Rightarrow$ GRAPH IS CONCAVE DOWN AT $X = 0.65$

3. (8 points) Given a function f and its first and second derivatives, f' and f'', which would you use and how would you use it to determine

(a) if f has a critical point at $x = 3$?

USE f' AND FIND WHERE $f'(x) = 0$ OR $f'(x)$ DNE

(b) the zeros of f?

USE f AND SOLVE $f(x) = 0$

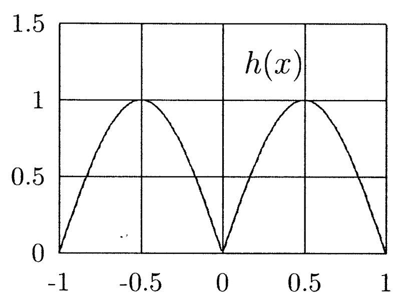
(c) if the graph of f has an inflection point at $x = -1$?

USE f'' AND FIND IF THE SIGN OF f'' CHANGES AT $X = -1$

(d) intervals on which f is decreasing?

USE f' AND FIND WHERE $f'(x) < 0$

4. (3 points) The function h, defined on the interval $[-1, 1]$, has the graph shown below. Find the critical numbers of h.



$h'(x) = 0$ WHEN $X = 0.5$ OR $X = -0.5$

$h'(x)$ DNE WHEN $X = 0$

CRIT NUMBERS ARE $X = 0.5$,
$X = -0.5$,
$X = 0$.

5. (20 points) The function g and its first two derivatives are shown below.

$$g(x) = \frac{4x^2 - 1}{2x^2 + 1}$$

$$g'(x) = \frac{12x}{(2x^2+1)^2} \qquad\qquad g''(x) = \frac{12 - 72x^2}{(2x^2+1)^3}$$

(a) Find all vertical and horizontal asymptotes of the graph of g.

SINCE $2x^2+1$ IS NEVER ZERO, THERE ARE NO POSSIBLE VERTICAL ASYMPTOTES.

$\boxed{\text{No V.A.'s}}$

$$\lim_{x \to \pm\infty} \frac{4x^2-1}{2x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to \pm\infty} \frac{4 - \frac{1}{x^2}}{2 + \frac{1}{x^2}} = \frac{4}{2} = 2$$

$\boxed{\text{H.A. } y = 2}$

(b) Use the 1st derivative test to find open intervals on which g is increasing/decreasing.

$g'(x) = 0 \Rightarrow x = 0$

$g'(x)$ DNE NEVER

SIGNS OF g': $-$ (left of 0), $+$ (right of 0)

$\boxed{\begin{array}{ll}\text{INCREASING ON} & (0, \infty) \\ \text{DECREASING ON} & (-\infty, 0)\end{array}}$

(c) Find all relative extreme values of g.

$\boxed{g(0) = -1 \text{ IS A RELATIVE MIN.}}$

(d) Use the 2nd derivative test to find open intervals on which the graph of g is concave up/down.

$g''(x) = 0 \Rightarrow 12 = 72x^2$

$\Rightarrow x = \pm\sqrt{\frac{12}{72}} = \pm \frac{1}{\sqrt{6}}$

$g''(x)$ DNE NEVER

SIGNS OF g'': $-$ (left of $-\frac{1}{\sqrt{6}}$), $+$ (between $-\frac{1}{\sqrt{6}}$ and $\frac{1}{\sqrt{6}}$), $-$ (right of $\frac{1}{\sqrt{6}}$)

$\boxed{\begin{array}{ll}\text{C.U. ON} & \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \\ \text{C.D. ON} & \left(-\infty, -\frac{1}{\sqrt{6}}\right) \cup \left(\frac{1}{\sqrt{6}}, \infty\right)\end{array}}$

(e) Find all points of inflection of the graph of g.

$g\left(\frac{1}{\sqrt{6}}\right) = -\frac{1}{4}$

$g\left(-\frac{1}{\sqrt{6}}\right) = -\frac{1}{4}$

INFLECTION POINTS ARE $\left(\frac{1}{\sqrt{6}}, -\frac{1}{4}\right)$ AND $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{4}\right)$

6. (5 points) Determine the following limit.

$$\lim_{x\to\infty} \frac{5x^4 - 8x^3 + 9x}{8x^7 - 99x^4 + 100} \cdot \frac{\frac{1}{x^7}}{\frac{1}{x^7}}$$

$$= \lim_{x\to\infty} \frac{\frac{5}{x^3} - \frac{8}{x^4} + \frac{9}{x^6}}{8 - \frac{99}{x^3} + \frac{100}{x^7}} = \frac{0}{8} = \underline{\underline{0}}$$

7. (8 points) Determine the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{3x^6 + 6x^2 + 8}}{-8x^3 + 7x^2 + 5x} \cdot \frac{\frac{1}{\sqrt{x^6}}}{\frac{1}{x^3}}$$

$\left(\sqrt{x^6} = x^3 \text{ FOR } x > 0 \right)$

Is the limit any different as $x \to -\infty$?

$$= \lim_{x\to\infty} \frac{\sqrt{3 + \frac{6}{x^4} + \frac{8}{x^6}}}{-8 + \frac{7}{x} + \frac{5}{x^2}} = \underline{\underline{\frac{\sqrt{3}}{-8}}}$$

YES, SINCE $\sqrt{x^6} = -x^3$ FOR $x < 0$,

THE LIMIT AS $x \to -\infty$

IS $\frac{\sqrt{3}}{8}$.

8. (8 points) Use Newton's method to approximate the only real solution of the equation $7 - 2x - x^3 = 0$. Write down the recursive formula for x_{n+1}, choose a suitable initial estimate x_0, and write down each improved estimate until your estimates differ by less than 0.00001.

$f(x) = 7 - 2x - x^3$

$f'(x) = -2 - 3x^2$

$$X_{N+1} = X_N - \frac{f(X_N)}{f'(X_N)}$$

Using $X_0 = 1$, we get

$X_1 = 1.8$

$X_2 = 1.592491468$

$X_3 = 1.569220697$

$X_4 = 1.568946441$

$X_5 = 1.568946403$

9. (8 points) Find the linearization $L(x)$ of the function $f(x) = \sin 2x + \cos x$ at $x = 0$. Then complete the following table. Round each entry to the nearest hundredth.

x	$f(x)$	$L(x)$
0.01	1.02	1.02
0.05	1.10	1.10
0.1	1.19	1.20

$L(x) = f(0) + f'(0)(x - 0)$

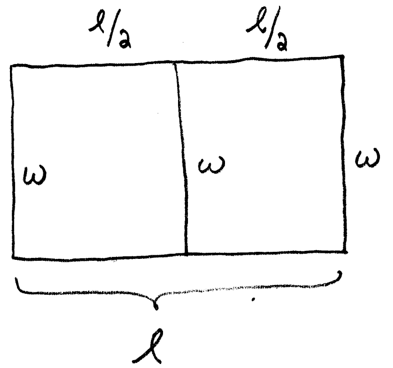
$f(0) = \sin(0) + \cos(0) = 1$

$f'(x) = 2\cos(2x) - \sin x$

$f'(0) = 2\cos(0) - \sin(0) = 2$

$\boxed{L(x) = 1 + 2x}$

10. (10 points) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



MAXIMIZE $A = \ell w$

SUBJECT TO $2\ell + 3w = 200$

$$\ell = \frac{200 - 3w}{2} = 100 - \frac{3}{2}w$$

$$A(w) = \left(100 - \frac{3}{2}w\right)w$$

$$= 100w - \frac{3}{2}w^2$$

DOMAIN: $0 \le w \le \frac{200}{3}$

MAXIMIZE

$$A(w) = 100w - \frac{3}{2}w^2 \text{ ON } \left[0, \frac{200}{3}\right]$$

$$A'(w) = 100 - 3w = 0 \Rightarrow w = \frac{100}{3}$$

CHECKING CRITICAL NUMBERS AND ENDPOINTS ...

$A(0) = 0$

$A\left(\frac{100}{3}\right) = \frac{5000}{3} \longleftarrow$ ABS MAX

$A\left(\frac{200}{3}\right) = 0$

DIMENSIONS ENCLOSING MAX AREA ARE

$w = \frac{100}{3}$ FT, $\ell = 50$ FT

Math 171 - Test 3b

November 15, 2012

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THESE PROBLEMS.

1. (10 points) Use algebra and calculus techniques to sketch the graph of

$$y = \frac{-x^3 + 3x^2 + 3x - 9}{10}.$$

Use graph paper! This problem is open-ended on purpose. Be thorough enough to make your work worth 10 points.
Free graph paper is available online at http://www.printfreegraphpaper.com.

$$y = -\frac{1}{10}x^3 + \frac{3}{10}x^2 + \frac{3}{10}x - \frac{9}{10} = -\frac{(x-3)(x^2-3)}{10}$$

NEGATIVE LEADING COEFF

$\Rightarrow$ GRAPH GOES DOWN ON RIGHT, UP ON LEFT

$y = 0 \Rightarrow x = 3,\ x = \pm\sqrt{3}$

1ST DERIVATIVE:

$$\frac{dy}{dx} = -\frac{3}{10}x^2 + \frac{6}{10}x + \frac{3}{10}$$

$$= -\frac{3}{10}(x^2 - 2x - 1) = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

SIGNS OF dy/dx: $-$ on $(-\infty, 1-\sqrt{2})$, $+$ on $(1-\sqrt{2}, 1+\sqrt{2})$, $-$ on $(1+\sqrt{2}, \infty)$

DECREASING ON $(-\infty, 1-\sqrt{2}) \cup (1+\sqrt{2}, \infty)$

INCREASING ON $(1-\sqrt{2}, 1+\sqrt{2})$

$x = 1-\sqrt{2}$, $y = -0.965685$ REL MIN

$x = 1+\sqrt{2}$, $y = 0.165685$ REL MAX

2ND DERIVATIVE:

$$\frac{d^2y}{dx^2} = -\frac{6}{10}x + \frac{6}{10} = 0$$

$$\Rightarrow x = 1$$

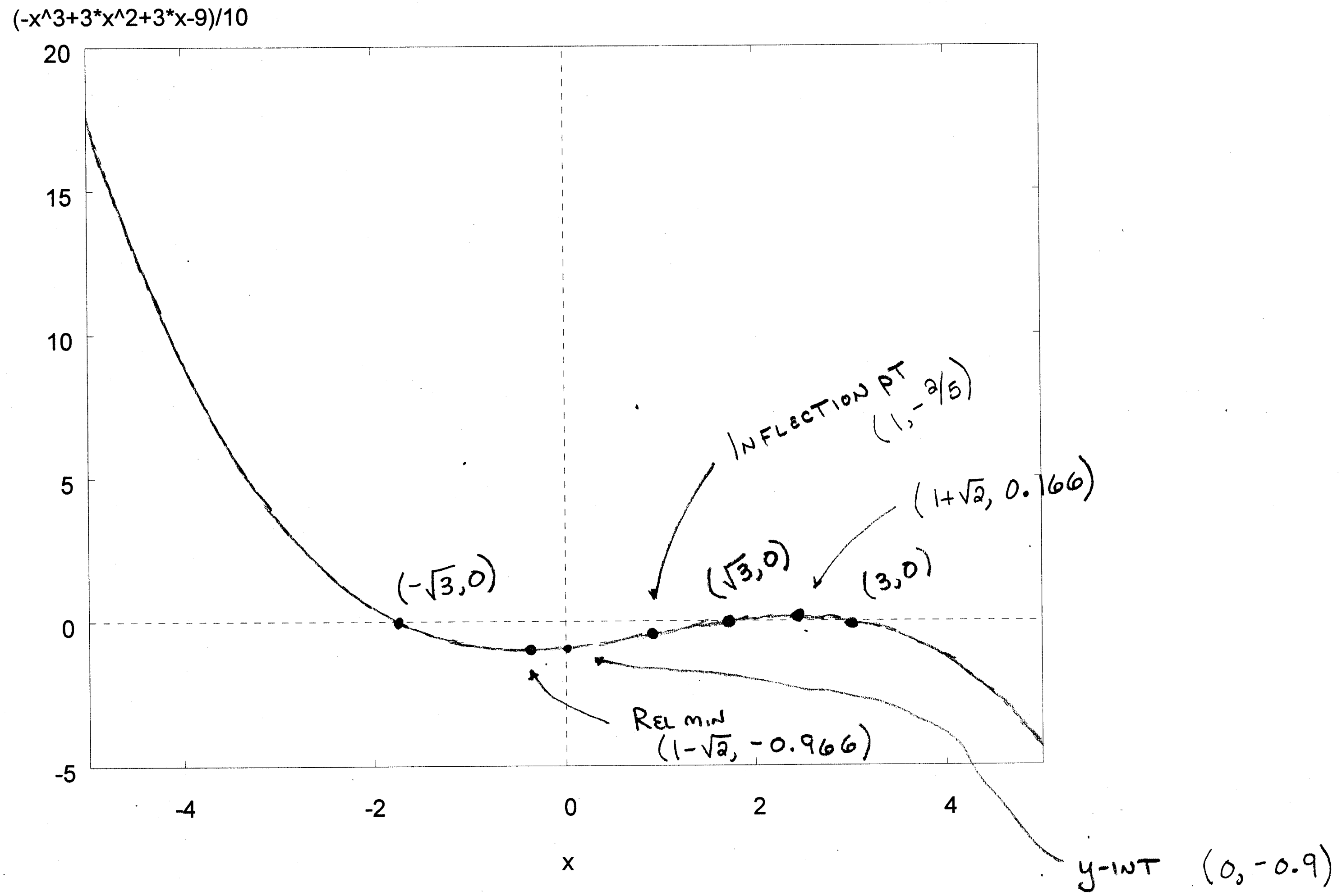
SIGNS OF d^2y/dx^2: $+$ on $(-\infty, 1)$, $-$ on $(1, \infty)$

GRAPH IS CU ON $(-\infty, 1)$

GRAPH IS CD ON $(1, \infty)$

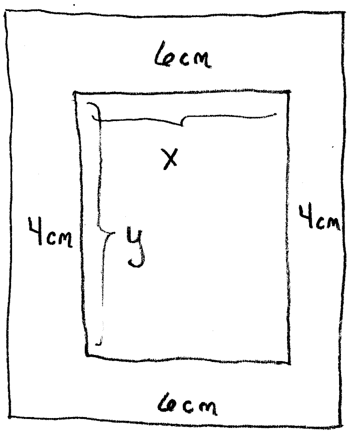
INFLECTION PT IS $(1, -2/5)$

SEE ATTACHED GRAPH



(-x^3+3*x^2+3*x-9)/10
20
15
10
5
0
-5
-4
-2
0
2
4
x
INFLECTION PT
(1, -2/5)
(1+√2, 0.166)
(-√3,0)
(√3,0)
(3,0)
REL MIN
(1-√2, -0.966)
y-INT (0, -0.9)

2. (10 points) The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm^2, find dimensions of the poster with the smallest total area.



Minimize $A = (x+8)(y+12)$

s.t. $xy = 384$

$$y = \frac{384}{x}$$

$$A(x) = (x+8)\left(\frac{384}{x} + 12\right), \quad x > 0$$

$$A(x) = 384 + \frac{3072}{x} + 12x + 96, \quad x > 0$$

$$A'(x) = -\frac{3072}{x^2} + 12 = 0$$

$$\Rightarrow \quad 12 = \frac{3072}{x^2}$$

$$\Rightarrow \quad x^2 = \frac{3072}{12} = 256$$

$$\Rightarrow \quad x = 16 \text{ cm}$$

Signs of A': $-$ on $(0, 16)$, $+$ on $(16, \infty)$

$x = 16$ gives a min

$x = 16$ cm

$y = \frac{384}{16} = 24$cm

This gives dimensions

$\boxed{24 \text{ cm by } 36 \text{ cm}}$